Passive Electrical Model of Silicon Photomultipliers

Kristen A. Wangerin, Gin-Chung Wang, Chang Kim, Yaron Danon

Abstract—An electrical model is developed to simulate, characterize, and predict the response of SSPM detectors for different device geometries and measurement circuit configurations. In particular, the model allows investigation of the effects of increasing parasitic capacitance with increasing diode area on the timing and magnitude of the readout signal. Passive components in the model are extracted from measurements and then used in the model to understand and predict device performance. The avalanche is represented with a switch in series with a voltage source and diode resistor, instead of a current source. This approach allows the change in potential, current through the diode, and timing of the avalanche to be simulated. Experimental and modeled pulses are compared for two different size devices. The model is first developed and validated using the 1x1 mm² device. Predictive capability is demonstrated with the 3x3 mm² device; in the scaling-up of the devices, only expected model parameters are changed, and the experimental and modeled pulses are in good agreement. The current through the diode and voltage change across the diode as functions of time agree with expectations.

I. INTRODUCTION

Solid-state photomultipliers (SSPMs) are a rapidly developing detector technology, with the potential to advance a range of applications, ranging from high-energy physics, biological sensors, nuclear medicine, DNA sequencing, and homeland security [1]. Currently, detectors are composed of a scintillator coupled to a photomultiplier tube (PMT). The scintillator converts gamma radiation into optical photons, and the PMT converts the optical photons to an electronic signal and amplifies it. The detector, however, is limited by the quantum efficiency of the photocathode of the PMT, and the bulky, fragile vacuum tube requires high voltage. Standard PMTs are also sensitive to magnetic fields, which distort electron trajectories to the first and second dynodes, and can be damaged by excess ambient light [2,3].

SSPMs overcome many of the limitations of PMTs while combining advantages of PMTs and silicon detectors. They are compact, robust, stable, and low power devices. Unlike PMTs, SSPMs are unaffected by magnetic fields, making them an attractive option for applications such as PET-MRI. SSPMs have high detection efficiency, high gain, and good energy resolution. Disadvantages are that they are currently small and expensive.

A SSPM is a photosensor consisting of an array of photodiodes, or cells, that are connected in parallel and operated above their breakdown voltage in Geiger mode [4,5]. When an optical photon strikes one of these cells, the cell undergoes an electrical breakdown, and its charge is collected onto a common electrode. The diode is in series with a quenching resistor, and the voltage across the diode drops during the avalanche. The decreasing potential slows the avalanche until the current is quenched and the cell begins to recharge. The output signal of the SSPM is proportional to the number of cells that are struck by optical photons.

An electrical model of an SSPM can enhance understanding of the design and behavior of SSPMs. The model in this work is developed from two previous electrical models that have been developed to simulate the behavior of solid-state photomultipliers [6,7]. Fig 1 shows the model developed by Pavlov et al [6], and Fig 2 shows the model developed by Corsi et al [7]. In both models, the cells of the SSPM are divided into one cell that undergoes breakdown and the remaining cells represented as an equivalent circuit. The time constants of the response of a cell undergoing breakdown and recharging are modeled with passive components. The avalanche of a Geiger cell is simulated using a current pulse in the diode.

The model by Pavlov et al. simulated the pixel current over time in a 1.1 x 1.1 mm² SSPM to characterize the timing of the devices and define dead time and maximum count rate. Using circuit parameters, the charge released from the diode was calculated, and the timing was estimated based on time constants of the circuit. The work simulated two readout resistors of 1000 and 50 Ω and concludes that the larger resistor slows down both the release of charge and recovery time of the pixel. There was a small inductor with a value of 10 nH/cm, presumably to account for stray inductions of the device.

The purpose of the model developed by Corsi et al. was to optimize the front-end readout with an SSPM coupled to an ideal and finite bandwidth preamplifier. A current source was again used to simulate the diode avalanche, and the charge was calculated based on model parameters of overvoltage and capacitance. There was an additional parasitic grid capacitance to correctly account for all time constants that characterize the shape of the signal waveform. A method for extraction of model parameters was described, and parameters are extracted for two unknown size SSPMs, one from ITC-irst and one from Photonique. The governing equations, which define the influence of parameters on the circuit time constants, were developed. The time constants of the circuit

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K. A. Wangerin is with GE Global Research, Niskayuna, NY 12189 USA (telephone: 518-387-5414, e-mail: wangerin@research.ge.com).
G-C. Wang is with GE Global Research, Niskayuna, NY 12189 USA (telephone: 518-387-7971, e-mail: wang@research.ge.com).
C. Kim is with GE Global Research, Niskayuna, NY 12189 USA (telephone: 518-387-4683, e-mail: kim@research.ge.com).
Y. Danon is with Rensselaer Polytechnic Institute, Troy, NY 12180 USA (telephone: 518-276-4008, e-mail: danony@rpi.edu).
were explored for readout resistor values of 20, 50, and 75 Ω as voltage versus time. Finally, the responses of the experimental data and model for one of the SSPPMs were compared using two different readout amplifiers, and the comparison plots are shown.

![SSPM model with a current pulse simulating the avalanche](image1)

In order to better relate the model to the physics, previous models are further developed in this work to simulate the avalanche with a switch, diode resistor, and voltage source, shown in Fig 14. The switch allows the potential across and current through the diode to be simulated based on the circuit and not the definition of the current source. Closing the switch initiates the avalanche. The potential decreases across the cell to the breakdown voltage. The current rises within a nanosecond and then decreases to the avalanche quenching current when the switch opens.

Experimental and modeled pulses are compared for 1x1 mm² and 3x3 mm² 25 μm devices over a range of bias voltages. Parameters in the model are extracted from experimental data for the two devices, and only these parameters are modified when simulating different devices. Modeled results are in good agreement in experimental results. The model will be able to predict the response of SSPPMs for different device geometries and measurement circuit configurations.

II. THEORY

SSPMs are composed of an array of avalanche photodiode (APD) cells operated in Geiger mode. APDs convert light directly into charge. When an optical photon is absorbed in the semiconductor material, an electron-hole pair is created. The detector is biased above breakdown, and the voltage creates a large electric field that accelerates the charge carriers toward the cathode and anode. As these carriers gain kinetic energy, they interact, liberating additional carriers. A cascading avalanche amplifies what would otherwise be a small signal from a single detected photon. The final output signal is large enough to be digitized by electronics with a high signal-to-noise ratio. The avalanche is stopped by a quenching resistor; the large resistance limits the current available to flow through the diode to sustain the avalanche current. The output signal is thus limited by the discharge of the potential built up across the capacitance of the silicon.

The potential and current across the diode as a function of time is shown in Fig 3. As the charge built up across the diode discharges, this potential decreases, and the probability of electron-hole pair generation decreases. The avalanche current stops increasing when the space charge in the depletion region from generated electron-hole pairs collapses the voltage such that the internal field is below the avalanche field. The potential drops to the breakdown voltage, $V_{br}$, shown in Fig 3, which is the voltage when an avalanche of zero current through the diode can be sustained, or the multiplication factor of electron-hole pair generation equals unity [9]. The breakdown voltage is better described as a band or region, rather than a rigid voltage point. The diode current drops, and when the current is on the order of 10-20 μA, the fluctuation of carriers will drop to zero, and the avalanche will be quenched [10].

The full discharge of one APD does not provide any information on the incoming energy of the photon, so the APD is subdivided into an APD cell array. The output detector signal is proportional to the number of APD cells that fire as a result of absorption of an optical photon. Fig 4 shows an equivalent circuit of the diodes and quenching resistors connected in parallel. The diodes are reversed biased, and the output signal is read as a voltage across a resistor to ground.

![SSPM array of pixels connected in parallel](image2)
A. Pulse Time Constants

The generation and transport of the charge carriers from the depletion region to the terminals of the device define the rise time of the current pulse. Details on the timing of the avalanche current can be found in [10,12,13,14].

The resistances and capacitances of the diode determine the recovery time. After the avalanche, the quenching capacitance recharges almost instantaneously; the potential across the diode is very close to zero, this capacitance is small, and it is in parallel with the very large quenching resistance. Thus, the recharging of the diode is first dominated by that of the quenching RC. The diode capacitance then begins to recharge, defining the pulse tail. As the voltage builds up across the diode, current flows through the quenching resistor, and the quenching capacitor will slowly discharge as the voltage across it changes.

B. Equivalent Electrical Model

An SSPM array can be represented as an electrical circuit [6]. The main electrical components are the quenching resistor (R_q), silicon resistor (R_t), and capacitor (C_d) of the diode. There is also a parasitic capacitance associated with the quenching resistor (C_q). Other stray inductance and capacitance of the diode are not explicitly considered, as these values are small [15].

Circuit parameters, such as the collected charge, diode capacitances, breakdown voltage, and quenching resistance, can be extracted. The current pulse from the diode can be considered a Dirac delta pulse in time with charge, Q, that can be expressed as

\[ Q = (V_{bias} - V_{br})(C_d + C_q) . \]  

This relationship holds true because the time constants of the circuit, dominated by the capacitance, are longer than those of the avalanche. The charge associated with the discharge of a single cell is proportional to the gain as a function of bias voltage. The charge generated in an avalanche is the time integral of the current, and current is the voltage measured over a resistor. Plotting bias voltage versus the charge, the slope of the curve provides \( C_d + C_q \). Extrapolating to the y-intercept provides the breakdown voltage, the point where the gain would equal zero [16]. These capacitances act in parallel.

The quenching capacitance is immediately recharged due to the large \( R_t \). Once \( C_q \) is fully recharged, \( C_q \) begins to recharge. As the voltage across \( C_q \) changes, \( C_q \) will discharge in parallel with \( C_d \).

The quenching resistor value can be extracted by forward biasing the diode [7]. The slope of the forward IV curve is equal to the transconductance. Taking the reciprocal gives the total resistance, \( R_{CLtot} \), which is equal to

\[ R_{CLtot} = \frac{R_q}{n} , \]  

where \( R_q \) is the quenching resistance of one cell, and \( n \) is the number of cells.

III. METHODS

Experimental and modeled pulses are compared over a range of bias voltages for 1x1 mm^2 and 3x3 mm^2 Hamamatsu Multi-Pixel Photon Counter (MPPC) devices with 25 mm cell size, shown in Fig 5. A NanoLED 05A laser initiates the avalanche in the SSPM cells, and the intensity is filtered such that only a few cells fire for each event. Pulse waveforms are acquired with a Tektronix TDS5104B digital phosphor oscilloscope. The bandwidth is 1 GHz with a sampling rate of 1 GS/s. It is assumed that the laser pulse is significantly shorter than the time response of the SSPM and that the frequency is low enough that the SSPM recovers fully between events. All measurements are performed with the SSPM in a light-tight aluminum box to reduce noise. A 1-mm diameter hole in the box allows the laser to shine on the SSPM. All data is taken at room temperature.

The 1x1 mm^2 devices, with pulse amplitudes on the order of millivolts, are characterized without a preamplifier. The 3x3 mm^2 devices are characterized with a Mini-Circuits ZX-60 4016E preamplifier. Due to their larger diode area and higher capacitance, these devices have more dark noise and smaller cell pulse amplitudes, in the range of 50 to 200 μV. The intrinsic noise of the oscilloscope is 1 mV. The true signal amplitude for the bigger devices is obtained by dividing the data taken with the preamplifier by the preamplifier gain. Because the preamplifier slightly shapes the signal, the pulse shape is obtained by increasing the laser intensity so that the signal is above the baseline noise without using the preamplifier. The measurement circuit connection diagram is shown in Fig 6.

The maximum amplitude of each pulse is histogrammed. The histogram has peaks and valleys according to the number of cells that fired for each event. The spacing of the peaks defines the cell gain. Data is taken over a range of voltages from near the breakdown voltage to a voltage when spontaneously breakdown begins to dominate. The optimum bias voltage is defined as that with the maximum signal-to-noise ratio and before excessive spontaneous breakdown.

Experimental data are used to extract model parameters as well as to help develop and validate the electrical model. The electrical model is developed in SPICE to first simulate the 1x1 mm^2 SSPM response in both pulse shape and amplitude. Each cell is composed of resistance, capacitance, and a switch. The quenching resistor is in parallel with the quenching capacitance. A capacitor simulates the charge buildup across the silicon diode and is in parallel with the diode resistor. The avalanche is modeled using a switch; the switch closes to initiate the avalanche and opens to end it and allow the cell to recharge. When the switch closes, the diode capacitor acts in series with the diode resistance and a voltage source that represents the breakdown potential. The time when the switch closes is determined by the current through the diode. The model is then compared to 3x3 mm^2 SSPM measurements to both validate the model and understand the effects of increasing SSPM size on pulse shape.
IV. RESULTS

A. 1x1 mm$^2$ SSPM Devices

1) Experimental Results

Experimental data is acquired over a range of applied voltages. For each bias voltage, digital waveforms are captured, shown in Fig 7 at a voltage of 71.9 V. The histogram of the maximum pulse amplitudes is shown in Fig 8. The histogram shows the number of cells firing for each event. As the bias voltage is increased, the pulse amplitude increases both because more cells are firing and the cell gain is increasing. From the histogram, events with the same number of firing cells are grouped and averaged to form a single pulse. Fig 9 shows the averaged pulses for each group. The tails of the pulses appear to increase for increasing bias voltage, and this is an effect of afterpulsing in the waveforms, a release of traps in the multiplication region that can retrigger avalanche breakdown [17]. Fig 10 shows the averaged pulses over a range of bias voltages.

2) Parameter Extraction

The breakdown voltage is extracted by plotting the gain of the diode versus voltage; it is the x-intercept of a linear fit to the data, when the gain of the diode would theoretically be zero. The gain or charge is calculated by integrating the signal over time and dividing by the equivalent resistance. The charge is first integrated over the entire signal, and results are shown in Fig 11; the calculated breakdown voltage of 69.9 V is overestimated due to afterpulsing. Fig 12 shows the breakdown voltage is 68.3 V when extracted integrating only the charge over the rise time of the pulse.

The quenching and diode capacitances are extracted using

\[ C = \frac{Q}{V_{\text{bias}} - V_{\text{breakdown}}} = \frac{69.7}{71.9 - 68.3} = 19.4 \, \text{pF}, \]

where \( Q \) is 69.7 ± 33.1 C for a bias voltage of 71.9 V, and \( V_{\text{breakdown}} \) is taken to be 68.3 V. The large uncertainty is due to the large pulse-to-pulse variation for small numbers of cells firing. The capacitance of \( C_{\text{quenching}} + C_{\text{diode}} \) is calculated to be 19.4 pF. These parameters can not be extracted as readily independently, so there is a degree of freedom on what their values are in the model. The diode capacitance is taken to be 25 pF, shown when validating the model, and the quoted terminal capacitance is 35 fF [18]. Some variation is expected between devices due to variations in silicon wafers.

The SSPM is forward biased to extract the quenching resistance. The current versus the voltage is plotted in Fig 11. The quenching resistance is equal to the slope of the linear region. For the 1 mm$^2$ diode and 25 µm cell size, there are 1600 cells. Using

\[ R_{\text{q, total}} = \frac{1}{R_{\text{q}}} = \frac{1}{0.0088} = 113.6 \, \Omega, \]

where \( R_{\text{q, total}} = 1/0.0088 = 113.6 \, \Omega \) and \( n = 1600 \), the quenching resistance, \( R_{\text{q}} \), of one cell is 181.8 kΩ.
3) Electrical Model Validation

Extracted parameters for quenching resistance and capacitance and diode capacitance are input into the model, shown in Fig 14. The breakdown voltage is taken to be 67.7 V. The model is validated by comparing the modeled pulse shapes to experimental data for events with three avalanching cells. The pulse shapes are compared through rise time, peak amplitude, and fall time, which are determined by the time constants of the circuit.

The change of voltage or potential across the diode is plotted, along with the current through the diode. During an avalanche, the voltage across the diode drops to the breakdown voltage, shown in Fig 15. As the potential decreases to the breakdown voltage, the number of charge carriers generated in the avalanche decreases, so the current also decreases, shown in Fig 16. When the probability to generate new carriers becomes small, the avalanche is no longer self-sustaining, and the current drops to zero. The switch then closes, and the cell begins to recharge. It is possible that the voltage across the diode may drop below the breakdown voltage, due to the space charge in the depletion region. This effect is not included in the modeling.

The effect of overvoltage on pulse shape and magnitude is investigated. The gain of the devices increases with overvoltage, shown in Fig 17(a). The voltage and current are shown in Fig 17(b). The modeled and experimental pulses are in good agreement at lower bias voltages, as shown for 71.1 V. As the voltage is increased, the tail of the experimental pulse is much higher; this is the result of after pulses in some of the pulses averaged and is not from the original avalanche event.

With constant switch timing, it is found that the amplitude of the modeled pulse does not increase as much as the experimental pulses do with increasing bias voltage. As the bias voltage increases, the potential across the diode is initially greater, so it takes longer for the current to drop to the quenching current. The switch timing is, therefore, modified so that the switch closes when the current reaches a constant value for all bias voltages. The quenching current is taken to be 25 uA, near previously reported quenching current values [10]. At higher bias voltages the total charge collected from the avalanche is greater, so the amplitude of the modeled pulses is increased and in better agreement with the experimental pulses, shown in Fig 18(a). The voltage across and current through the diode as a function of time are shown in Fig 18(b).
Fig 19. Pulse waveforms acquired with the preamplifier (70.0 V).

Fig 20. Pulse amplitude histogram with the preamplifier (70.0 V).

Fig 21. Averaged pulses with the preamplifier for 0 to 5 cells firing for a bias voltage of 71.0 V.

Fig 22. Averaged pulses with the preamplifier over a range of bias voltages.

Fig 23. Gain-corrected averaged pulses with the preamplifier over a range of bias voltages.

Fig 24. Averaged pulses (raw data) acquired without the preamplifier and with a strong laser pulse.

Fig 25. Averaged pulses acquired without the preamplifier normalized to gain-corrected preamplifier amplitudes.

B. 3x3 mm² SSPM Devices

1) Experimental Results

The 3x3 mm² SSPMs are tested at bias voltages from 69.0 to 71.0 V. Data is first taken with the preamplifier. The laser pulse is attenuated so that only a few cells fire for each event, shown in Fig 19. Fig 20 shows the histogram of the peak pulse amplitudes. Pulses are grouped and averaged in Fig 21 according to the number of cells firing for the event. Fig 22 shows averaged pulses with three cells firing for a range of bias voltages. The pulses are corrected by accounting for the gain of the preamplifier in Fig 23. The amplitude of the response from a single cell is on the order of hundreds of millivolts and is less for the larger device due to the parasitic capacitance of the other cells and a larger quenching resistor.

The preamplifier distorts the pulse shape, so data is also acquired without the preamplifier and with a stronger laser pulse. Fig 24 shows pulse waveforms for a range of bias voltages. The number of cells firing is not determinable. The waveforms for each bias voltage are averaged to obtain one pulse. The pulses are scaled in Fig 25 to the pulse amplitude of the gain-corrected preamplifier data.

2) Parameter Extraction

Integration of the current gives estimates of the breakdown voltage as well as the total cell capacitance. The amplitude corrected data that was taken without the preamplifier is integrated as voltage over time to get the total charge of the event. The variation in charge is smaller than what it was for the 1x1 mm² device because the original acquired waveforms are well above the noise. The afterpulsing is also too small to be seen. The charge versus the bias voltage is plotted in Fig 26, and the breakdown voltage is calculated to be 67.9 V.
The quenching and diode capacitances are extracted. The charge, \( Q \), is \( 52.3 \pm 2.2 \) C for a bias voltage of 70.5 V. The breakdown voltage is taken to be 67.6 V. The capacitance of \( C_q + C_d \) is calculated to be 18.0 pF, compared to 19.4 pF for the 1x1 mm\(^2\) device. The capacitances input into the model are kept equal to 5 pF for \( C_q \) and 25 pF for \( C_d \) from the 1x1 mm\(^2\) model, as it is expected that these values would be similar.

The diode is forward biased to extract the quenching resistance. Fig 27 shows the collected IV data and the curve fit to the linear portion. Multiplying the inverse slope by the number of cells gives the quenching resistance,

\[
R_q = \left( \frac{1}{\text{slope}} \right) \left( \frac{\text{diode size}}{\text{cell size}} \right)^2 = \left( \frac{1}{0.0577} \right) \left( \frac{3 \text{ mm}}{25 \mu \text{m}} \right)^2
\]

resulting in a value of 249.6 k\( \Omega \).

The electrical model is then tested against the 3x3 mm\(^2\) device. Relevant model parameters are modified to represent the values extracted from 3x3 mm\(^2\) data, shown in Fig 28. The quenching resistor is taken to be 250 k\( \Omega \), and the breakdown voltage is taken to be 67.6 V. The diode and quenching capacitances are kept constant. The pulse shapes and voltage and current of the diode are compared. Differences in pulse shape are considered and explored.

Fig 26. Total charge integrated for the averaged pulses over a range of bias voltages. Extrapolating the data to zero estimates the diode breakdown voltage.

Fig 27. Extraction of diode equivalent resistance by forward biasing the diode.

3) Electrical Model Validation

The SSPM is simulated with one cell firing, and the remaining 14399 cells are grouped as an equivalent circuit. The experimental data to which the modeled results are compared is the gain-normalized data acquired without the preamplifier. The modeled and experimental data are compared in Fig 29. The modeled pulse follows the shape of the experimental pulse except for some small differences in pulse shape that shift with bias voltage. For a voltage of 69.0 V, the pulses follow closely over the rise time and first part of the tail. The second part of the modeled pulse tail is slower than the experimental. As the bias voltage increases, the differences between the pulse shapes shift to the first parts of the pulse. For a voltage of 71.0 V, the modeled pulse has a slower rise time and a faster decay over the first part of the tail in comparison to experimental pulse. The second part of the pulse tails match well for all voltages. Unlike for the 1x1 mm\(^2\) data, the 3x3 mm\(^2\) does not show afterpulsing; there are many cells firing in each event so the after pulses from a few cells are not visible in the pulse.

The switch timing is varied so that it always closes when the current reaches the quenching current of 25 \( \mu \text{A} \). The voltage across the diode and the current through the diode are shown in Fig 30. The current reaches the quenching value before the asymptotic part of the curve, so the switch timing does not change much over the range of bias voltages; the switch opens at 2.12 ns at 69.0 V and 2.16 ns at 71.0 V.

Fig 28. Electrical model for 3x3 mm\(^2\) SSPM with calculated parameters for one firing cell and the remaining cells represented as an equivalent circuit.

Fig 29. Comparison of 3x3 mm\(^2\) diode experimental and modeled pulses.

Fig 30. Voltage across the diode and current through the diode.
The differences in pulse shape with bias voltage are explored. It is hypothesized that the bigger diode behaves differently due to extra capacitance and inductance. The experimental pulse shape may also be different from the equivalent electrical model of the SSPM is valid for small numbers of cells firing, and the modeled pulse shape is representative of experimental results. As the number of cells firing increases to hundreds or thousands, the differences in pulse shape increase. Although this is a limitation of the model, the differences found here are small.

V. CONCLUSIONS

An electrical model is developed in SPICE to simulate, validate, and predict the response of SSPMs and enable further understanding and development of these detectors. Passive components are used to simulate the time constants of the response. The avalanche is modeled using a voltage source and switch, which allows for more accurate and intuitive modeling of the device behavior. Model parameters are related to the physics of the device, and values are extracted based on experimental measurements. An RC circuit shields the effects of the voltage cable, and an LC circuit at the readout accounts for effects of the measurement setup.

The model is developed and validated using a 1x1 mm² device. Experimental and modeled pulses are compared. The switch timing in the model is adjusted with increasing bias voltage so that the switch always closes when the diode current reaches the latching current. The pulse amplitudes are in agreement for pulses at different bias voltages and with different numbers of cells firing. The pulse rise time and decay times are also in agreement. The decay times of the experimental pulses appear slower for higher bias voltages, but this is a result of afterpulsing in the diode and not from the response of a single cell.

A 3x3 mm² device is used to demonstrate the predictive capabilities of the model. The model parameters are updated to reflect those extracted from the 3x3 mm² device. All other parameters are kept constant. The experimental and modeled pulses are mostly in agreement. Small differences in pulse rise time, amplitude, and fall time are hypothesized to be the effect of larger diode area as well as the result of the variation in number of cells firing on time constants. With the validated model, the properties of the readout signal of an SSPM can be investigated as a function of bias voltage, readout circuit, and device design and geometry.

REFERENCES