(H1) Logic Tools

Selmer Bringsjord
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1 Arguments

- An **argument** is sequence $\phi_1, \phi_2, \phi_3, \ldots \phi_n$ of declarative sentences (or **propositions**); the final member ϕ_n is said to be the **conclusion**, and the remaining members are said to be the **premises**.
- An argument A is **formally valid** iff it's impossible for the premises of A to be true while the conclusion of A is false.
- An argument A is **veracious** iff all of its premises are true.
- An argument A is **sound** iff A is both formally valid and veracious.

2 Rules of Inference

In moving from ϕ_k to ϕ_{k+1} in an argument it is often useful to use rules of inference guaranteed to preserve truth in the transition. Here are a few commonly used valid rules of inference:

modus ponens from ϕ, $\phi \rightarrow \psi$ infer to ψ

modus tollens from $\neg\psi$, $\phi \rightarrow \psi$ infer to $\neg\phi$

hypothetical syllogism from $\phi \rightarrow \psi$, $\psi \rightarrow \delta$ infer to $\phi \rightarrow \delta$

Here are some commonly used fallacious or invalid rules of inference (make sure you see why they're invalid):

denying the antecedent from $\neg\phi$, $\phi \rightarrow \psi$ infer to $\neg\psi$

affirming the consequent from ψ, $\phi \rightarrow \psi$ infer to ϕ

3 The Propositional Calculus

The alphabet of the propositional calculus (the simplest formal language useful in representing arguments to come) contains an infinte supply of propositional variables (used to stand for propositions) $p_0, p_1, p_2, \ldots$ and the truth functional connectives $\wedge$ (and), $\vee$ (or), $\neg$ (not), $\rightarrow$ (if ...,then ...), $\leftrightarrow$ (if and only if). And grammatically correct expressions (= well formed formulas, or just wffs) in the propositional calculus are exactly those that conform to the following grammar:

- if p is a propositional variable then it's a wff
- if ϕ and ψ are wffs then $\phi * \psi$ is a wff, where $* = \neg, \wedge, \vee, \rightarrow, \leftrightarrow$.

4 First Order Logic (FOL)

FOL is obtained by adding to the propositional calculus the quantifiers $\exists$ ("there exists an") and $\forall$ ("for all"), along with corresponding formation rules and rules of inference. For example:

(a formation rule) if ϕ is a wff then $\forall x \phi$ is a wff.

universal instantiation – a rule of inference from $\forall x \phi$ infer to $\phi(a/x)$, where $\phi(a/x)$ is the result of substituting the constant (you can think of a constant for now as a proper name) a for every occurrence of the variable x in ϕ.

Arguments presented in this class will sometimes by formalized in FOL.

5 Modal Logic

Some different senses of necessity and possibility:

1. It's necessary that you renew your driver's license before it expires.
2. Food, shelter, and clothing are necessary.
3. (laws of nature, e.g. Boyle's law)
4. It's not possible for a human to swim across the Atlantic
5. It's necessary that either today is Monday or today is not Monday.
6. It's possible for a human to swim across the Atlantic.

logical possibility $\Diamond \phi$, where ϕ is a wff of FOL or the prop. calc.

logical necessity $\Box \phi$, where ϕ is a wff of FOL or the prop. calc.

Some basic modal logic:

- $\Box\phi$ iff $\neg\Diamond\neg\phi$
- $\Box\phi \rightarrow \phi$
- $\phi \rightarrow \Diamond\phi$

And, finally, a principle that I urge you to affirm:

If ϕ is conceivable then $\Diamond\phi$.