Example (cont.): c) Parametric Representation - Notes

- Positive Normal Convex Fuzzy Numbers form a commutative Semi-group
- Fuzzy numbers do not have the inverse element for sum and multiplication, as in groups:
  - **m + (-m) = 0**
  - **m x (1/m) = 1**
- For example, let's define the fuzzy number

$$\tilde{m} = (a, b, \alpha, \beta)$$

Example (cont.): c) Parametric Representation - Notes

- Then, we have:

Eq. (1)

$$\tilde{m} + (-\tilde{m}) = (a, b, \alpha, \beta) + (-b, -a, \beta, \alpha) =$$

$$= (a - b, b - a, \alpha + \beta, \alpha + \beta)$$

Eq. (5)

Eq. (2)

$$\tilde{m} \times (\frac{1}{\tilde{m}}) = (a, b, \alpha, \beta) \times \left(\frac{1}{b}, \frac{1}{a}, \frac{\beta}{b(b+\beta)}, \frac{\alpha}{a(a-\alpha)} \right)$$

$$= \left(\frac{a}{b}, \frac{b}{a}, \frac{a\beta + b\alpha}{b(b+\beta)}, \frac{a\beta + b\alpha}{a(a-\alpha)} \right) \text{ if } \tilde{m} > 0$$

Eq. (7)

$$= \left(\frac{b}{a}, \frac{a}{b}, \frac{-b\alpha - a\beta}{a(a-\alpha)}, \frac{-a\beta - b\alpha}{b(b+\beta)} \right) \text{ if } \tilde{m} < 0$$

Eq. (10)

Example (cont.): c) Parametric Representation - Notes

- The condition $\tilde{m} > 0$ means that the **support** of the fuzzy number is a positive interval, i.e.:

$$(a - \alpha) > 0$$

- Similarly $\tilde{m} < 0$ means that the **support** of the fuzzy number is a negative interval, i.e.:

$$(b + \beta) < 0$$

Example (cont.): c) Parametric Representation - Notes

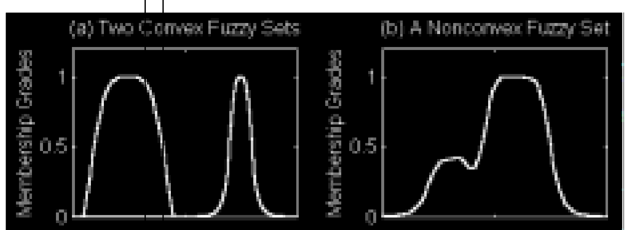
- Equations (1), (4), and (5) in the table are exact formulae.
- All other equations are approximations since the shape of the resulting slope is not linear - but we are approximating them as linear
- All conditions for formulae applicability refer to the sign of the support (as in previous slide)

Convexity of Fuzzy Sets

A fuzzy set *A* is convex if for any λ in [0, 1],

$$\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_A(x_1), \mu_A(x_2))$$

Alternatively, *A* is convex is all its α-cuts are convex.



convexmf.m

Normality of Fuzzy Sets

A fuzzy set *A* is *normal* if

$$Height(A) = Max_x A(x) = 1$$