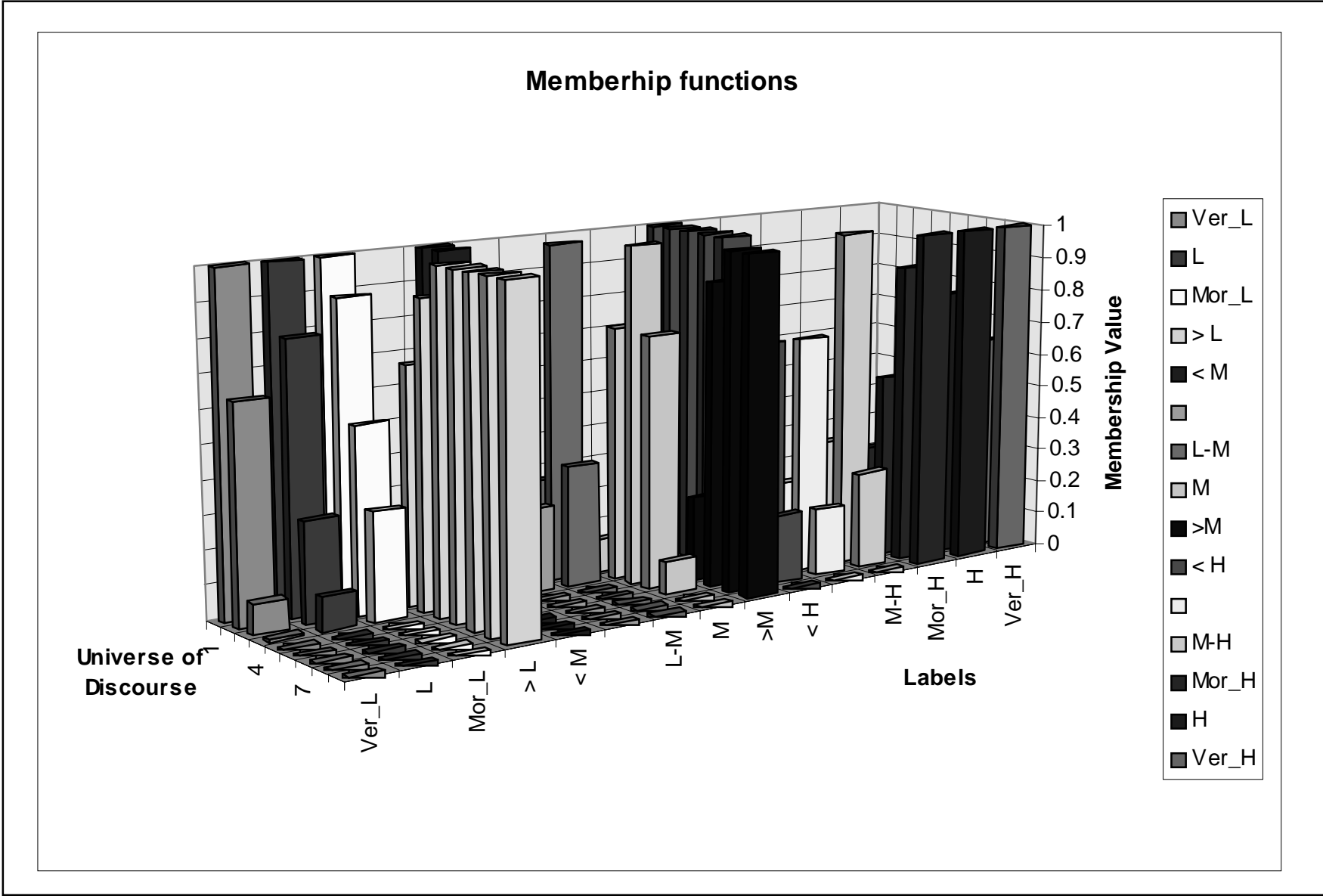


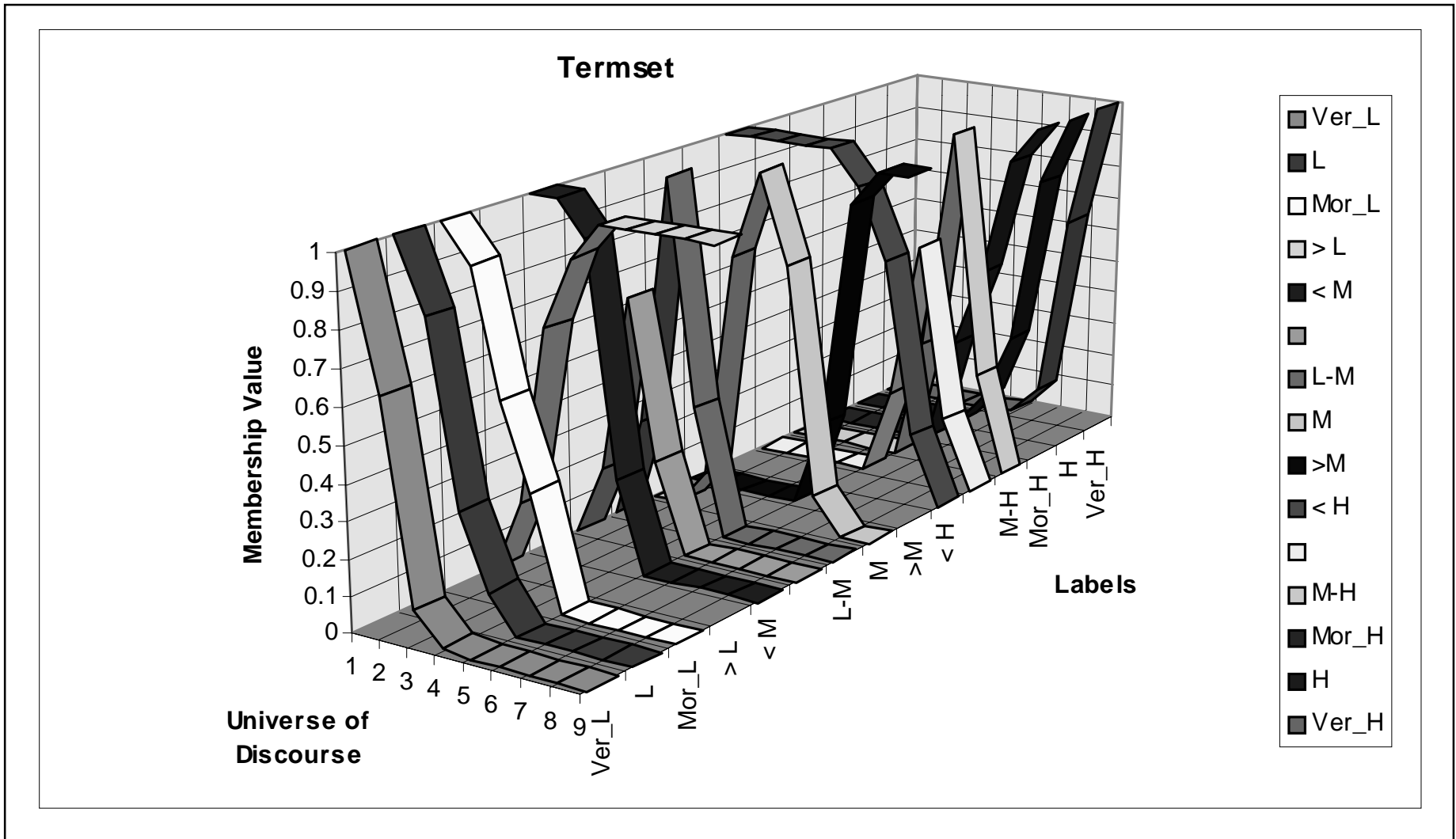
Linguistic Terms (Termset)

Label	Symbol	Universe of Discourse								
		1	2	3	4	5	6	7	8	9
Very Low	Ver_L	1	0.64	0.09	0.01	0	0	0	0	0
Low	L	1	0.80	0.30	0.10	0	0	0	0	0
More or less Low	Mor_L	1	0.89	0.55	0.32	0	0	0	0	0
Above Low	>L	0	0.20	0.70	0.90	1	1	1	1	1
Below Medium	<M	1	1.00	0.90	0.25	0	0	0	0	0
Above Low and Below Medium		0	0.20	0.70	0.25	0	0	0	0	0
Between Medium and High	L-M	0	0.29	1	0.36	0	0	0	0	0
Medium	M	0	0	0.10	0.75	1	0.75	0.10	0	0
Above Medium	>M	0	0	0	0	0	0.25	0.90	1	1
Below High	<H	1	1	1	1	1	0.90	0.70	0.20	0
Above Medium and Below High		0	0	0	0	0	0.25	0.70	0.20	0
Between Medium and High	M-H	0	0	0	0	0	0.36	1	0.29	0
More or less High	Mor_H	0	0	0	0	0	0.32	0.55	0.89	1
High	H	0	0	0	0	0	0.10	0.30	0.80	1
Very High	Ver_H	0	0	0	0	0	0.01	0.09	0.64	1

Termset Plot



Termset Plot (Interpolated)



Example of Fuzzy Relation R & Modus Ponens

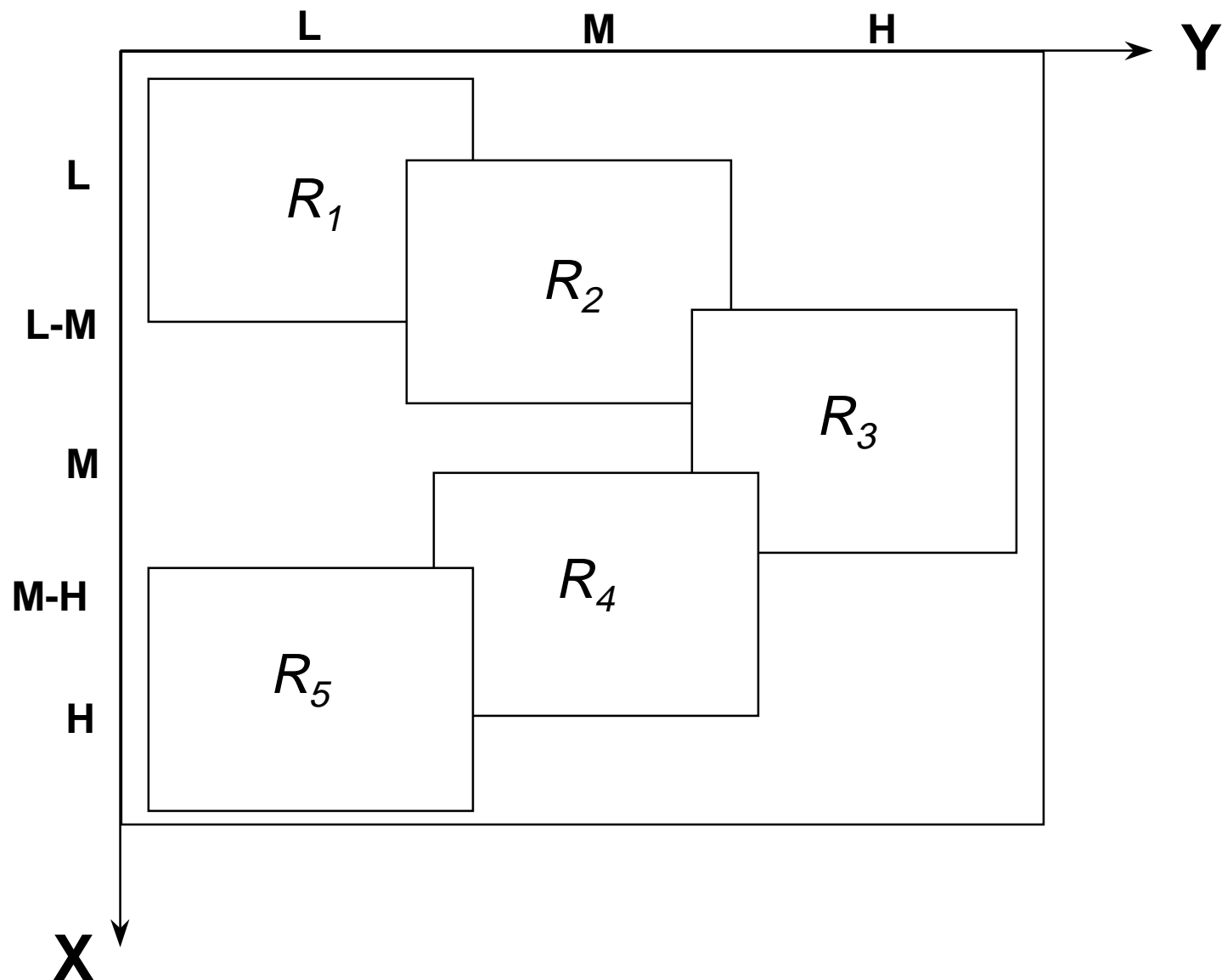
Label	Symbol	Universe of Discourse								
		1	2	3	4	5	6	7	8	9
Very Low	Ver_L	1	0.64	0.09	0.01	0	0	0	0	0
Low	L	1	0.80	0.30	0.10	0	0	0	0	0
Between Low and Medium	L-M	1	0.80	0	0.10	0	0	0	0	0
Medium	M	0	0	0.10	0.75	1	0.75	0.10	0	0
Between Medium and High	MH	0	0	0.1	0.75	1	0.75	0	0.00	0
High	H	0	0	0	0	0	0.10	0.30	0.80	1

$$R_i: [X \times Y] \rightarrow [0,1]$$

$$R = \bigcup_{i=1}^5 R_i$$

R	X	Y
R_1	L	L
R_1	L-M	M
R_3	M	H
R_4	M-H	M
R_5	H	Ver_L

Corresponding Fuzzy Graph



Modus Ponens Implementation: Three Methods

Method 1

- a) Relation R is the Union of the Cartesian Products
- b) Cylindrical Extension of Input I
- c) Intersection with Relation R
- d) Projection of Intersection

Method 2

- a) Relation R is the Union of the Cartesian Products
- b) Inner Product $\langle Min, Max \rangle$ between Input I and Relation R

Method 3

- a) λ_i = degree of rule_i applicability (T-norm of Possibility Measures between Input I and rule_i LHS)
- b) Weigh rule_i RHS using λ_i as weights
- c) Union of Weighted Outputs

Method 1a: Union of Fuzzy Relations

R_1		Y = L								
		1	0.8	0.3	0.1	0	0	0	0	0
L = X	1	1.0	0.8	0.3	0.1	0.0	0.0	0.0	0.0	0.0
	0.8	0.8	0.8	0.3	0.1	0.0	0.0	0.0	0.0	0.0
	0.3	0.3	0.3	0.3	0.1	0.0	0.0	0.0	0.0	0.0
	0.1	0.1	0.1	0.1	0.1	0.0	0.0	0.0	0.0	0.0
	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

$$\begin{aligned}
 R_1(x,y) &= \\
 &= [\text{Low}(x) \times \text{Low}(y)] \\
 &= \text{Min}\{\text{Low}(x), \text{Low}(y)\}
 \end{aligned}$$

R_2		Y = M								
		0	0	0.1	0.75	1	0.8	0.1	0	0
M = L = X	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.3	0.0	0.0	0.1	0.3	0.3	0.3	0.1	0.0	0.0
	1	0.0	0.0	0.1	0.8	1.0	0.8	0.1	0.0	0.0
	0.4	0.0	0.0	0.1	0.4	0.4	0.4	0.1	0.0	0.0
	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

$$\begin{aligned}
 R_2(x,y) &= \\
 &= [\text{Low-Medium}(x) \times \text{Medium}(y)] \\
 &= \text{Min}\{\text{Low-Medium}(x), \text{Medium}(y)\}
 \end{aligned}$$

Method 1a: Union of Fuzzy Relations (cont.)

R_3		Y = H									
		0	0	0	0	0	0.1	0.3	0.8	1	
M = X	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.1	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.1
	0.8	0.0	0.0	0.0	0.0	0.0	0.1	0.3	0.8	0.8	0.8
	1	0.0	0.0	0.0	0.0	0.0	0.1	0.3	0.8	1.0	1.0
	0.8	0.0	0.0	0.0	0.0	0.0	0.1	0.3	0.8	0.8	0.8
	0.1	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.1	0.1	0.1
	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	

$$\begin{aligned}
 R_3(x,y) &= \\
 &= [\text{Medium}(x) \times \text{High}(y)] \\
 &= \text{Min}\{ \text{Medium}(x), \text{High}(y) \}
 \end{aligned}$$

R_4		Y = M									
		0	0	0.1	0.8	1	0.8	0.1	0	0	
H = M = X	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	
	0.4	0.0	0.0	0.1	0.4	0.4	0.4	0.1	0.0	0.0	
	1	0.0	0.0	0.1	0.8	1.0	0.8	0.1	0.0	0.0	
	0.3	0.0	0.0	0.1	0.3	0.3	0.3	0.1	0.0	0.0	
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0		

$$\begin{aligned}
 R_4(x,y) &= \\
 &= [\text{Medium-High}(x) \times \text{Medium}(y)] \\
 &= \text{Min}\{ \text{Medium-High}(x), \text{Medium}(y) \}
 \end{aligned}$$

Method 1a: Union of Fuzzy Relations (cont.)

R_5		Y = Ver_L								
		1	0.64	0.09	0.01	0	0	0	0	0
H = X	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	0.1	0.10	0.10	0.09	0.01	0.0	0.0	0.0	0.0	0.0
	0.3	0.30	0.30	0.09	0.01	0.0	0.0	0.0	0.0	0.0
	0.8	0.80	0.64	0.09	0.01	0.0	0.0	0.0	0.0	0.0
1	1.00	0.64	0.09	0.01	0.0	0.0	0.0	0.0	0.0	

$$\begin{aligned}
 R_5(x,y) &= \\
 &= [\text{High}(x) \times \text{VeryLow}(y)] \\
 &= \text{Min}\{ \text{High}(x), \text{VeryLow}(y) \}
 \end{aligned}$$

R		Y								
		X	1.00	0.80	0.30	0.10	0.0	0.0	0.0	0.0
0.80	0.80		0.30	0.29	0.29	0.29	0.10	0.0	0.0	
0.30	0.30		0.30	0.75	1.00	0.75	0.10	0.10	0.10	
0.10	0.10		0.10	0.36	0.36	0.36	0.30	0.75	0.75	
0.00	0.00		0.00	0.00	0.00	0.10	0.30	0.80	1.00	
0.10	0.10		0.10	0.36	0.36	0.36	0.30	0.75	0.75	
0.30	0.30		0.10	0.75	1.00	0.75	0.10	0.10	0.10	
0.80	0.64		0.10	0.29	0.29	0.29	0.10	0.0	0.0	
1.00	0.64		0.09	0.01	0.0	0.0	0.0	0.0	0.0	

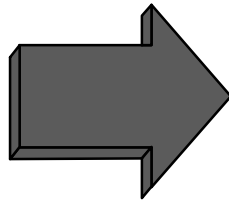
$$R = \bigcup_{i=1}^5 R_i$$

Method 1b: Input Cylindrical Extension

$$\text{Example: } Y_1 = X_1 \circ R$$

Cylindrical Extension

1
0.64
0.09
0.01
0
0
0
0
0



1	1	1	1	1	1	1	1	1	1
0.64	0.64	0.64	0.64	0.64	0.64	0.64	0.64	0.64	0.64
0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09
0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$X_1 = \text{Very Low}$

$\bar{X}_1 = \text{Cylindrical Extension(Very Low)}$

Method 1c: Intersection of Cyl. Ext. with Relation

Intersection

Cylindrical Extension

1	1	1	1	1	1	1	1	1
0.64	0.64	0.64	0.64	0.64	0.64	0.64	0.64	0.64
0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09
0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

Relation

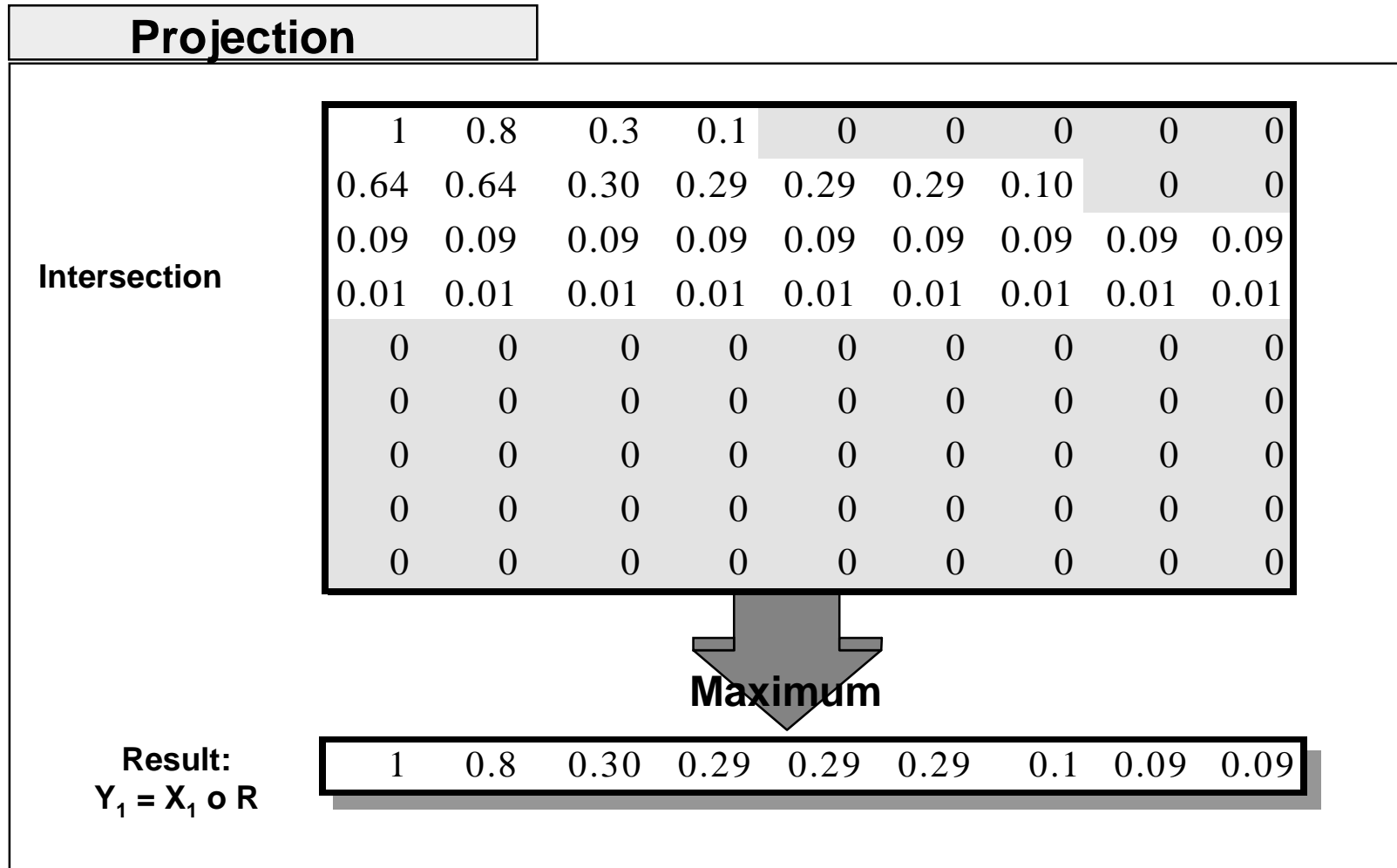
1	0.8	0.3	0.1	0	0	0	0	0
0.8	0.8	0.30	0.29	0.29	0.29	0.10	0	0
0.3	0.3	0.30	0.75	1	0.75	0.10	0.10	0.10
0.1	0.1	0.10	0.36	0.36	0.36	0.30	0.75	0.75
0	0	0	0	0	0.1	0.30	0.80	1
0.1	0.10	0.10	0.36	0.36	0.36	0.30	0.75	0.75
0.3	0.30	0.10	0.75	1	0.75	0.10	0.10	0.10
0.8	0.64	0.10	0.29	0.29	0.29	0.1	0	0
1	0.64	0.09	0.01	0	0	0	0	0

Minimum (pointwise)

Intersection

1	0.8	0.3	0.1	0	0	0	0	0
0.64	0.64	0.30	0.29	0.29	0.29	0.10	0	0
0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09
0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

Method 1d: Projection of Intersection



Method 2 b: Inner Product <Min Max>

Example: $Y_1 = X_1 \circ R$

Inner Product

$X^T_1 = \text{Very Low}$

1	0.64	0.09	0.01	0	0	0	0	0
---	------	------	------	---	---	---	---	---

R

1	0.8	0.3	0.1	0	0	0	0	0
0.8	0.8	0.30	0.29	0.29	0.29	0.10	0	0
0.3	0.3	0.30	0.75	1	0.75	0.10	0.10	0.10
0.1	0.1	0.10	0.36	0.36	0.36	0.30	0.75	0.75
0	0	0	0	0	0.1	0.30	0.80	1
0.1	0.10	0.10	0.36	0.36	0.36	0.30	0.75	0.75
0.3	0.30	0.10	0.75	1	0.75	0.10	0.10	0.10
0.8	0.64	0.10	0.29	0.29	0.29	0.1	0	0
1	0.64	0.09	0.01	0	0	0	0	0

<Min Max> Inner Product

[similar to Vector-Matrix Multiplication, using <Min Max> instead of <x,+>]

Result:
 $Y_1 = X_1 \circ R$

1	0.8	0.30	0.29	0.29	0.29	0.10	0.09	0.09
---	-----	------	------	------	------	------	------	------

Method 3 a: Degree of Rule Applicability (λ_i)

Example: $Y_1 = X_1 \circ R$

Degree of Rule Applicability

R	X	Y
R_1	L	L
R_1	L-M	M
R_3	M	H
R_4	M-H	M
R_5	H	Ver_L

Input: Very Low	1	0.6	0.09	0.01	0	0	0	0	0
X_1 : Low	1	0.8	0.3	0.1	0	0	0	0	0
Ver_L AND L	1	0.6	0.09	0.01	0	0	0	0	0

Max $\lambda_1 = 1$

Input: Very Low	1	0.64	0.09	0.01	0	0	0	0	0
X_2 : Low-Medium	0	0.3	1	0.36	0	0	0	0	0
Ver_L AND L-M	0	0.3	0.09	0.01	0	0	0	0	0

Max $\lambda_2 = 0.29$

Input: Very Low	1	0.6	0.09	0.01	0	0	0	0	0
X_3 : Medium	0	0	0.1	0.75	1	0.75	0.1	0	0
Ver_L AND M	0	0	0.09	0.01	0	0	0	0	0

Max $\lambda_3 = 0.09$

Method 3 a: Degree of Rule Applicability (λ_i)(cont.)

Example: $Y_1 = X_1 \circ R$

Degree of Rule Applicability

R	X	Y
R_1	L	L
R_1	L-M	M
R_3	M	H
R_4	M-H	M
R_5	(H)	Ver_L

Input: Very Low	1	0.6	0.09	0.01	0	0	0	0	0
X_4 : Medium-High	0	0	0	0	0	0.36	1	0.29	0
Ver_L AND M-H	0	0	0	0	0	0	0	0	0

Max

$$\lambda_4 = 0$$

Input: Very Low	1	0.6	0.09	0.01	0	0	0	0	0
X_5 : High	0	0	0	0	0	0.1	0.3	0.8	1
Ver_L AND H	0	0	0	0	0	0	0	0	0

Max

$$\lambda_5 = 0$$

Method 3 b: Weighting Rules Output

$$\text{Example: } Y_1 = X_1 \circ R$$

Weighting of Rules Output

R	X	Y
R_1	L	L
R_1	L-M	M
R_3	M	H
R_4	M-H	M
R_5	H	Ver_L

1	0.8	0.3	0.1	0	0	0	0	0
1	0.8	0.3	0.1	0	0	0	0	0

$$\lambda_1 = 1$$

Weighting (min)

0	0	0.10	0.75	1	0.75	0.10	0	0
0	0	0.10	0.29	0.29	0.29	0.10	0	0

$$\lambda_1 = 0.29$$

Weighting (min)

0	0	0	0	0	0.1	0.3	0.8	1
0	0	0	0	0	0.1	0.1	0.09	0.09

$$\lambda_1 = 0.09$$

Weighting (min)

0	0	0.1	0.75	1	0.75	0.1	0	0
0	0	0	0	0	0	0	0	0

$$\lambda_1 = 0$$

Weighting (min)

1	0.64	0.09	0.01	0	0	0	0	0
0	0	0	0	0	0	0	0	0

$$\lambda_1 = 0$$

Weighting (min)

Method 3 c: Union of weighted Outputs

Example: $Y_1 = X_1 \circ R$

Union of Weighted Outputs

Output of Rule r_i

r_1

1 0.8 0.3 0.1 0 0 0 0 0

r_2

0 0 0.10 0.29 0.29 0.29 0.10 0 0

r_3

0 0 0 0 0 0.1 0.1 0.09 0.09

r_4

0 0 0 0 0 0 0 0 0

r_5

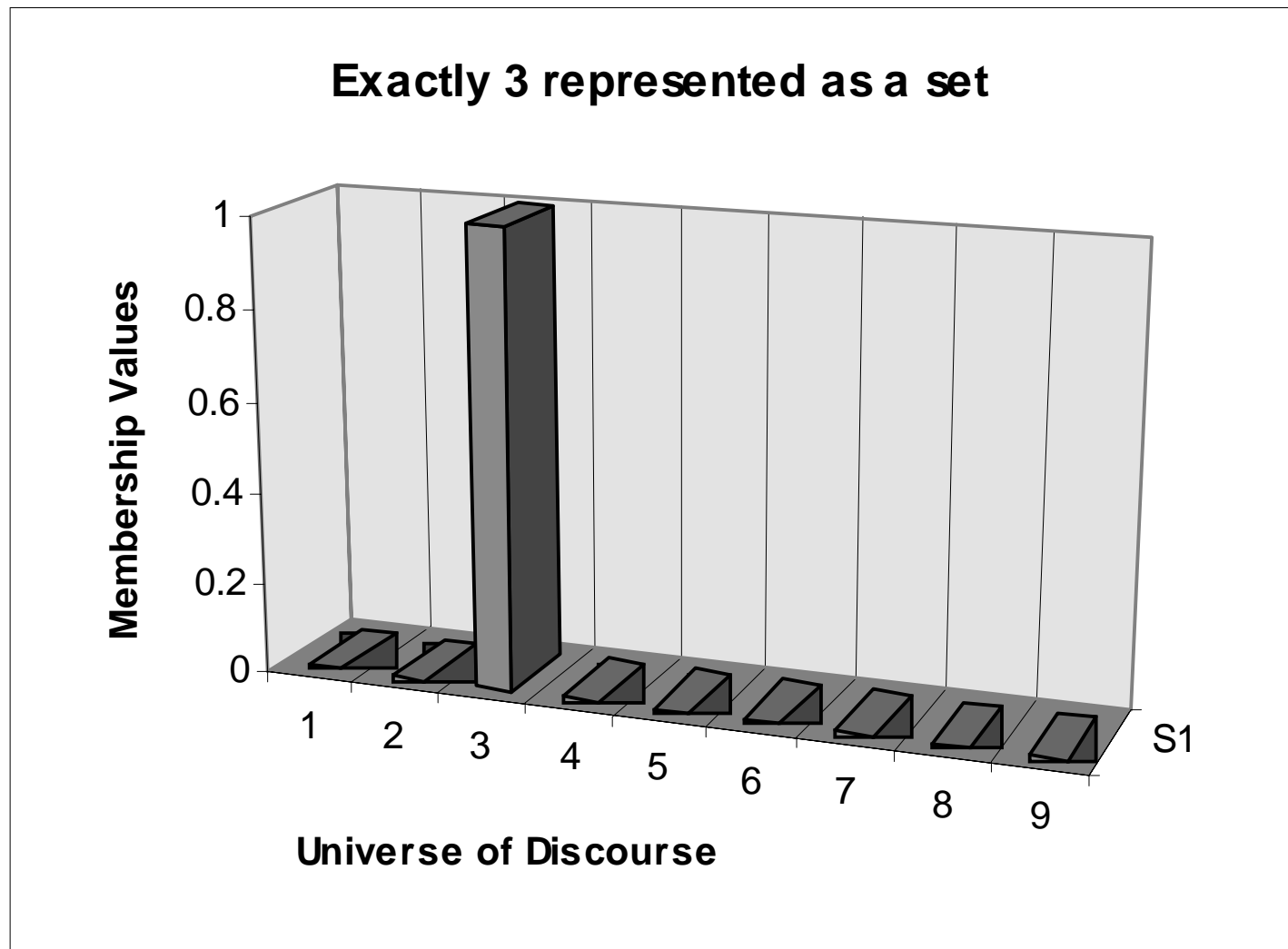
0 0 0 0 0 0 0 0 0

$$Output = \bigcup_{i=1}^5 Output(r_i)$$

1 0.8 0.3 0.29 0.29 0.29 0.10 0.09 0.09

Example with Crisp Input (Exactly 3)

Universe of Discourse	1	2	3	4	5	6	7	8	9
Exactly Three	0	0	1	0	0	0	0	0	0



Example with Crisp Input (Exactly 3)

Second Method

Input = Exactly 3

Exactly 3

Output = Exactly 3 \circ R_{TOT}

0 0 1 0 0 0 0 0 0 $\langle \text{min, max} \rangle$

Note:

$\langle \text{min, max} \rangle$ means the inner product using the min and max operator

This is similar to the vector-matrix multiplication, which is denoted by the inner product $\langle \cdot, + \rangle$

R_{TOT}									
	1	0.8	0.3	0.1	0	0	0	0	0
	0.8	0.8	0.3	0.29	0.29	0.29	0.1	0	0
	0.3	0.3	0.3	0.75	1	0.75	0.1	0.1	0.1
	0.1	0.1	0.1	0.36	0.36	0.36	0.3	0.75	0.75
	0	0	0	0	0	0.1	0.3	0.8	1
	0.1	0.1	0.1	0.36	0.36	0.36	0.3	0.75	0.75
	0.3	0.3	0.1	0.75	1	0.75	0.1	0.1	0.1
	0.8	0.64	0.1	0.29	0.29	0.29	0.1	0	0
	1	0.64	0.09	0.01	0	0	0	0	0

0.3 0.3 0.3 0.75 1 0.75 0.1 0.1 0.1

Analysis of Storage Complexity for Generalized Modus Ponens - Legend

Legend:	D Number of sampling of X_{ij} , i.e.: $ X_{ij} = D$, for $j=1, \dots, n$
	m Number of sampling for Y_i , i.e.: $ Y_i = m$, for $i=1, \dots, r$
	n Number of inputs, i.e.: $ X_i = n$, for $i=1, \dots, r$
	t Numbers of linguistic values of each state variable X_{ij}
	r Maximum numbers of rules in relation: $r = t^n$
	Ar Maximum number of active rules in relation : $A_r = 2^n$ (for each crisp input, assuming overlapping only among adjacent terms in termset)
	Rt Union of R_i relations (for $i=1, \dots, r$), i.e. $ R_t = m D^n$
	T Rule Table: $ T = r(nd+m)$

Analysis of Storage Complexity for Generalized Modus Ponens - Table 1 (D=11)

**For D = m =11
(Sampling Points)**

Inputs	Values	Rules	Active Rules	Size of Cartesian Product	Size of Rule Table
n	t	$r= t^n$	$A_r= 2^n$	$ Rt = m D^n$	$ T = (nD+m)r$
2	3	9	4	1,331	297
2	5	25	4	1,331	825
2	7	49	4	1,331	1,617
2	9	81	4	1,331	2,673
3	3	27	8	14,641	1,188
3	5	125	8	14,641	5,500
3	7	343	8	14,641	15,092
3	9	729	8	14,641	32,076
4	3	81	16	161,051	4,455
4	5	625	16	161,051	34,375
4	7	2401	16	161,051	132,055
4	9	6561	16	161,051	360,855
5	3	243	32	1,771,561	16,038
5	5	3125	32	1,771,561	206,250
5	7	16807	32	1,771,561	1,109,262
5	9	59049	32	1,771,561	3,897,234
6	3	729	64	19,487,171	56,133
6	5	15625	64	19,487,171	1,203,125
6	7	117649	64	19,487,171	9,058,973
6	9	531441	64	19,487,171	40,920,957

Analysis of Storage Complexity for Generalized Modus Ponens - Table 2 (D=21)

For D = m =21
(Sampling Points)

Inputs	Values	Rules	Active Rules	Size of Cartesian Product	Size of Rule Table
n	t	$r = t^n$	$A_r = 2^n$	$ Rt = m D^n$	$ T = (nD+m)r$
2	3	9	4	9,261	567
2	5	25	4	9,261	1,575
2	7	49	4	9,261	3,087
2	9	81	4	9,261	5,103
3	3	27	8	194,481	2,268
3	5	125	8	194,481	10,500
3	7	343	8	194,481	28,812
3	9	729	8	194,481	61,236
4	3	81	16	4,084,101	8,505
4	5	625	16	4,084,101	65,625
4	7	2401	16	4,084,101	252,105
4	9	6561	16	4,084,101	688,905
5	3	243	32	85,766,121	30,618
5	5	3125	32	85,766,121	393,750
5	7	16807	32	85,766,121	2,117,682
5	9	59049	32	85,766,121	7,440,174
6	3	729	64	1,801,088,541	107,163
6	5	15625	64	1,801,088,541	2,296,875
6	7	117649	64	1,801,088,541	17,294,403
6	9	531441	64	1,801,088,541	78,121,827