Nonlinear Programming
In Class Problems 9/9/03

1. Let \( A \) be an \( m \times n \) real matrix, \( b \) a real \( m \) and \( c \) a real \( n \) vector. Consider the primal and dual LPs.

\[
\begin{align*}
\text{(Primal)} & \quad \min_y \quad c'y \\
&s.t. \quad A'y = b \\
y & \geq 0
\end{align*}
\]

\[
\begin{align*}
\text{(Dual)} & \quad \max_x \quad b'x \\
&s.t. \quad Ax \leq c
\end{align*}
\]

(a) Assume that the primal LP is feasible. Show that the dual LP is infeasible if and only if the primal is unbounded.

(b) Assume that the dual LP is feasible. Show that the primal LP is infeasible if and only if the dual is unbounded.

Note this is two parts of the famous tricotomy theorem for linear programming.

2. Prove Gordan’s Theorem of the Alternative using Farkas’ Theorem.

Either

\[
Ax > 0 \quad (2)
\]

or

\[
A'y = 0, \quad y \geq 0, \quad y = 0 \quad (3)
\]

has a solution but not both.