Nonlinear Programming
Homework 4

Homework is due at the beginning of class on Tuesday, October 31. Only problems 1 and 4 need to be handed in. The rest are practice.

1. Consider the following problem:

\[
\begin{align*}
\text{max}_x & \quad x_1^2 + 4x_1x_2 + x_2^2 \\
\text{s.t.} & \quad x_1 + x_2 \leq 8 \\
& \quad -x_1 + 2x_2 \leq 4 \\
& \quad x_1, x_2 \geq 0
\end{align*}
\]

(a) Using the KKT conditions, find an optimal solution to the problem.
(b) Test for the second-order optimality conditions.
(c) Does the problem have a unique optimal solution?

2. Consider the following problem:

\[
\begin{align*}
\text{min} & \quad (x_1 - \frac{9}{4})^2 + (x_2 - 2)^2 \\
\text{s.t.} & \quad x_1 + x_2 \leq 6 \\
& \quad x_2 - x_1^2 \geq 0 \\
& \quad x_1, x_2 \geq 0
\end{align*}
\]

(a) Write the KKT optimality conditions and verify that these conditions are true at the point \( \bar{x} = (\frac{3}{2}, \frac{7}{4})^t \).
(b) Interpret the KKT conditions graphically.
(c) Verify Second order necessary and sufficient conditions.
(d) Show that \( \bar{x} \) is indeed the unique optimal solution.

3. Consider the following problem, where \( a_j, \ b, \) and \( c_j \) are positive constants:

\[
\begin{align*}
\text{min}_x & \quad \sum_{j=1}^n \frac{c_j}{x_j} \\
\text{s.t.} & \quad \sum_{j=1}^n a_j x_j = b \\
& \quad x_j \geq 0, \quad j = 1, \ldots, n.
\end{align*}
\]

Write the KKT conditions, and solve for the point \( (\bar{x}, \bar{u}) \) satisfying these conditions. Is this point (locally) optimal? establish your claim.