# Computational Optimization

Laboratory 6 - 3/07/08 - Matlab Optimization Toolbox In class assignment

The Matlab Optimization Toolbox contains many of the optimization routines discussed in this class. It has routines for both unconstrained and constrained optimization. Today we are going to look at a routine for constrained minimization called "fmincon". See

`http://www.mathworks.com/access/helpdesk/help/toolbox/optim/fmincon.html`

for a description. These are all ideas that we will be investigating in class. "fmincon" can optimize nonlinear functions with nonlinear constraints. But today we will just be looking at the case of minimizing nonlinear functions with linear equality constraints.

Let's use it the function subject to linear constraints:

$$\begin{array}{ll} \min_{x_1,x_2,x_3} & x_1^4 + 2x_2^4 - 3x_3^2 \\ \textit{subject to} & x_1 + x_2 + x_3 = 3 \end{array}$$

1. First make an M-file (f6.m) that returns the function, gradient and the Hessian depending on how many arguments it is asked to return. Or download from course webpage.

```
function [f,g,h] = f6(x)
  f = x(1)^4+2*x(2)^4+-3*x(3)^2;
  if nargout > 1
     g = [4*x(1)^3, 8*x(2)^3,-6*x(3)];
  end;
  if nargout > 2
     h = [12*x(1)^2 0 0; 0 24*x(2)^2 0;  0 0 -6];
  end;
```

2. Invoke the optimization routine with 'f6' and the constraint Ax=b with starting point x0:

```
x0 = [3 0 0]';
Aeq=[1 1 1 ];
beq=[3]'
x = fmincon(@f6,x0,[],[],Aeq,beq)
```

   You should get that the optimal objective value is -102.7441.

3. We can look at various values returned by fmincon.

```
[ xmin,fval, exitflag, output, lambda, grad]= fmincon(@f6,x0,[],[],Aeq,beq)
```

   Exitflag lets you know the termination condition. Output contains information about the algorithm used and how things went. Look at output. You should see that the method converges in about 13 function evaluations.

4. You can get the Hessian and gradient at xmin by typing

   ```
   [f g h] = f6(xmin)
   ```

   Observe that the gradient at xmin is not 0.

5. Let's check the first order necessary optimality conditions. The optimal Lagrangian multipliers for the equality constraints are in lambda.eqlin. So we know the first order conditions are satisfied if the following quantities are about 0.

   ```
   Aeq*xmin-beq
   g + Aeq'*lambda.eqlin
   ```

   Are they? From the class notes, the FONC are $Aeq(xmin)-beq = 0$ and $\nabla f(xmin) - A * \lambda = 0$ and here we checked $Aeq(xmin)-beq = 0$ and $\nabla f(xmin) + A * \lambda = 0$. But this is okay because we can always take $\lambda = -\lambda$ for equality constraints. You have to be careful about this when looking at the Lagrangian multipliers for equalities returned by any particular solver, since it depends on the convention used for writing the KKT conditions for equality constraints.

6. Use the Null command in matlab to compute the nullspace of Aeq.

   ```
   Z=Null(Aeq);
   ```

   Verify that the alternative form of the FONC are satisified namely that $Z'\nabla f(x) = 0$ and $Aeq * x = beq$.

7. Is the hessian positive definite at xmin? Is the function convex?

8. The SONC are satisfied if the FONC hold at $xmin$ and $Z'\nabla^2 f(xmin)Z$ is p.s.d.. Does $x$ satisfy the SONC to be a local minima?

9. The SOSC are satisfied if the FONC hold at $xmin$ and $Z'\nabla^2 f(xmin)Z$ is p.d.. Can we conclude that SOSC are satisfied at $xmin$ and therefore $xmin$ is a strict local min?

10. Repeat this exercise for

$$\begin{array}{ll} \min_x & \frac{1}{2}x'C'x \\ s.t. & Ax = b \end{array} \tag{1}$$

   with

$$C = \begin{bmatrix} 0 & -13 & -6 & -3 \\ -13 & 23 & -9 & 3 \\ -6 & -9 & -12 & 1 \\ -3 & 3 & 1 & 3 \end{bmatrix} \quad A = \begin{bmatrix} 2 & 1 & 2 & 1 \\ 1 & 1 & 3 & -1 \end{bmatrix} \quad b = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \tag{2}$$