Computational Optimization

Exam

Name:

This exam is due 10 a.m. on Friday, May 9. You can either give the exam to me in my office por put it in an envelope in my mailbox anytime before then. If you have any trouble with the format of the exam or think there is some error, please email or ask me directly. Clarifications about the exam will be posted on the course webpage. All matlab codes used for question 2 are on the course webpage. All work should be done independently without help from any other human being. **To receive credit you must justify your answers**. Please include diaries and m files for any matlab codes written.

1. (10 pts) Problem 1, pg. 552. Linear and Nonlinear Programming by Nash and Sofer.

2. (10 pts) Consider the following problem

$$\begin{array}{ll} min_x & f(x_1, x_2) = e^{3x_1+4x_2} \\ s.t & g(x_1, x_2) = x_1^2 + x_2^2 = 1 \end{array} \tag{1}$$

Prove analytically that $x* = (-3/5, -4/5)'$ with $\lambda = -5/2e^{-5}$ is the optimal solution.

3. (20 pts) Goldstein and Price's function is defined as follows:

$$f(x_1, x_2) = (1 + (x_1 + x_2 + 1)^2(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2))*$$
$$(30 + (2x_1 - 3x_2)^2(18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2))$$

The diary of a Matlab run of the file runexp.m can be found on the course webpage. Steepest descent (sdopt), preconditioned conjugate gradient (pcgopt), and quasi-Newton with BFGS (bfgsopt) each with and without gradients supplied were applied, along with quasi-Newton with DFP (dfpopt) to Goldstein and Price's function.

(a) What starting point was used for each of the methods?

(b) Did each algorithm stop at an optimal solution? If not, which algorithms stopped at an optimal solution. Why did the other algorithms stop?

(c) Compare and discuss the first and second order optimality conditions upon termination of each algorithm.

(d) Compare and discuss the computation times used by the algorithms. You can assume the elapsed time provided by tic-toc is correct.

(e) Compare and discuss the objective function values upon termination of each algorithm.

(f) Which algorithm performed best and why?

(g) What might you do to improve these results?

4. (30 pts) Problem 11, Nash and Sofer pg 500. Hand in your m-files and diary of your run.

5. (30 pts) For graduate credit only. Consider the problem,

$$min_x f(x) \quad s.t. \quad g_i(x) \geq 0, \quad i = 1, \ldots, m \tag{2}$$

One strategy to solving this problem is to use a series of linear programs to compute suitable search directions. Suppose $x \in R^n$ is a feasible point with $g_i(x) = 0$ for $i \in I$. Suppose $g_i(x)$ is convex at x for each $i \in I$. Show that the following problem produces a feasible descent direction or concludes that x is a KKT point of the original problem (2). Discuss how this problem could be used as part of a feasible descent method.

For a fixed x:

$$\begin{array}{ll} min_d & \nabla f(x)'d \\ s.t. & \nabla g_i(x)'d \geq 0 \quad i \in I \\ & -1 \leq d_j \leq 1, \quad j = 1, \ldots, n \end{array} \tag{3}$$