NAME____

Computational Optimization Midterm

Exams are to be returned in class on March 7. If you must miss class, please put the exams in a sealed envelope (inter campus mail ones are fine) in my box in Amos Eaton 327 before 10 a.m. on March 7. Exams may be turned in early. Hand in copies of all Matlab files that you write. Late policy is -5% after 9 a.m. 3/7, and -20% on Monday 3/10. No late exams accepted after 3/10.

All work on the exam should be completed independently. You can email or come by and ask me questions. You may not consult other human beings. Answered questions/clarifications pertinent of the exam will be placed on the course webpage. Feel free to ask me questions about any materials in the text or covered in the class. References to NW refer to Numerical Optimization by Nocedal and Wright, 2nd edition, 2006.

I certify that I completed this exam independently without consulting any other human being other than the instructor Professor Bennett.

Signature ____________________________ date ______________________

1. **PART (a) UNDERGRADUATE + GRADUATE (5 points)**
   The BFGS Quasi-Newton algorithm performs rank-2 updates of the weighting matrix. Recall, we say the formula to update a matrix $B \in R^{n \times n}$ is rank-2 if there exists matrices $U \in R^{n \times 2}$ and $V \in R^{n \times 2}$ such that

   $$B_{k+1} = B_k + U_k V_k',$$

   or equivalently if there exists $u_{k1}, u_{k2} \in R^n$ (the columns of U) and $v_{k1}, v_{k2} \in R^n$ (the columns of V) such that

   $$B_{k+1} = B_k + u_{k1} v_{k1}' + u_{k2} v_{k2}'.$$

   The update formula for B_{k+1} (equation 6.19 in NW) is clearly in this form. But the update for H_{k+1}, the inverse of B_{k+1} (given in equation 6.17 of NW), is not obviously in rank-2 form. Construct the matrices $U \in R^{n \times 2}$ and $V \in R^{n \times 2}$ such that the update formula has the form

   $$H_{k+1} = H_k + U_k V_k'.$$

   PART (b) GRADUATE ONLY (5 points)
   Question 6.4 in N.W, Use the Sherman-Morrison formula (A.27) to show that (6.24) is the inverse of (6.25). For extra credit, do this for the BFGS equations as well.

2. Let $f(x_1, x_2) = 2x_1^2 + x_2^2 - 2x_1x_2 + 2x_1^3 + x_1^4$.

PART (a) UNDERGRADUATE and GRADUATE (5 points)
Suppose that f is minimized starting from x_o=(0,-2). Verify that p_0=(0,1)' is a direction of descent.

PART (b) UNDERGRADUATE and GRADUATE (5 points)
Suppose an exact step size is used along the direction p_0, e.g.
$\alpha_1 \in \arg\min f(x_0 + \alpha_0 p_0)$.
What is α_1, the size of the step taken?

PART (c) GRADUATE ONLY (5 points)
Suppose that a backtracking linesearch (Algorithm 3.1 in NW) is used to find a step length satisfying the Armijo Condition in the above problem. Does α_1 = 1 satisfy the sufficient decrease condition with c=.5? For what values of c does α_1 = 1 satisfy the Armijo Condition.

PART (d) GRADUATE ONLY (5 points)
Is this a (strictly) convex or concave function? Prove or disprove your claim.

3. UNDERGRADUATE and GRADUATE (25 points)
Download the routine newton2.m and modify it to do BFGS using weighting matrix H_{k+1} in equation (6.17). Use H_0 = the identity. Repeat the experiment in Lab 5 with the same settings and starting points except skip function 4. Complete Table A, attached, comparing Newton with BFGS.
Note that the format of Table A has slightly changed from that of Lab 5. The condition column is now changed to Min? Indicate yes if solution gave an acceptable solution or no if you think it failed. The routine newtonexact2.m is the same as exactnewton except the stopping criteria has been changed so it works for all f(x) and x, so the results will change for function 2.
Your final answer should include:

- Table A completed
- Copy of all new m files that you wrote.
- Discussion of results. If the min was not found, analyze why the routine failed and include this in your discuss of the results. Discuss the differences observed for Quasi-Newton and Newton's method and how this relates to the theory of these algorithms.

4. UNDERGRADUATE and GRADUATE (10 points)

Consider the function

$$\min\ f(x_1, x_2) = (x_2 - 2x_1)^2 + x_2^4 .$$

Find the minimizer of f. Answer these questions

- Are the second-order necessary conditions for a local minimizer satisfied at this point?
- Are the second-order sufficient conditions satisfied?
- Is this point a strict local minimizer?
- Is it a global minimizer?