A CELL-BASED MANY-TO-ONE DYNAMIC SYSTEM OPTIMAL MODEL AND ITS HEURISTIC SOLUTION METHOD FOR EMERGENCY EVACUATION

Henry X. Liu, Department of Civil Engineering, University of Minnesota, Twin Cities

Xiaozheng He, Department of Civil Engineering, University of Minnesota, Twin Cities

Jeff X. Ban, Institute of Transportation Studies, University of California, Berkeley

ABSTRACT

An efficient prescriptive dynamic traffic assignment model is critical for the effective traffic management under emergency evacuation. Although a number of dynamic traffic assignment models have been proposed in the previous studies, it is almost impossible to apply them for real-time emergency traffic management due to the high computational cost. However, for the real-time emergency traffic management, computational efficiency becomes essential but reasonably detailed representation of traffic flow dynamics has to be maintained. By assuming that all evacuees have only one destination, the safe area, the prescriptive model is established as a many-to-one dynamic system optimal problem and a linear program is formulated based on the cellular representation of traffic network. To achieve the required computational efficiency, instead of solving the linear program directly, we develop a heuristic algorithm called HASTE and provide a close approximation to the optimal solution. We apply the system optimal model to a hypothetical emergency evacuation scenario in the downtown of Minneapolis, Minnesota, and the solutions are generated both by the linear program solver and by HASTE. The comparisons between different solution approaches indicate that the heuristic can provide close-to-optimal evacuation strategies with a much higher computational efficiency, which is important to real-time evacuation operations.

1. INTRODUCTION AND MOTIVATION

Emergency evacuation, a mass movement of people from disaster-impacted areas to safer ones in a timely manner, has been studied and practiced for decades aiming to mitigate the disastrous consequences. It is undisputable that during evacuation, the ground transportation system plays a central role. The challenge is that how transportation networks can be better prepared in advance and managed in real time to deal with emergency evacuation in a logical and effective manner. Interestingly, existing evacuation research in transportation field has been mostly focusing on the planning stage, from various perspectives such as traffic management policies, origin-destination (O-D) trip estimations, and behavioral analysis. Moreover, due to the distinct features of different types of disasters, specific planning models have been developed for various evacuation scenarios, including nuclear plant crisis, hurricane, flooding, and fire, etc. For detailed discussions regarding evacuation planning models, we refer to reviews by Southworth (1991) and Urbina and Wolshon (2003).

While evacuation planning is important for the emergency preparedness, it hardly gives good predictions of future evacuation scenarios due to the highly dynamic and uncertain features involved in such extreme events. In an operational stage, the real-time feedback strategy can significantly enhance the evacuation performances due to the highly dynamic nature in evacuation. Recently, Liu *et al.* (2006) have developed an integrated framework for real time dynamic traffic management under emergency evacuation using model reference adaptive control (MRAC). In the MRAC framework, a short-term prescriptive dynamic traffic assignment (DTA) model is needed to provide the optimal traffic state for orderly evacuation. These desired traffic states then serve as a reference point and the difference between the system desired states and real-world observations can be compared, therefore traffic control strategies can be adjusted adaptively. This prescriptive DTA problem can be normally formulated as a dynamic system optimal (DSO) model so as to minimize the total fatalities and property losses during emergency evacuation.

To make the prescriptive model operational in real time, the model needs to be realistic in traffic flow modelling and can be solved efficiently. By adopting the concept of “super destination zone” proposed in Chiu et al. (2005), the prescriptive DTA model for emergency evacuation under no-notice disaster can be simplified as a many-to-one problem, where all vehicles have only one destination, the safe area. With this simplification, the first-in-first-out (FIFO) discipline is simply dependent on the departure time (or arrival time) as proven by Kuwahara and Akamatsu (1993) and Akamatsu (2000). In addition, the flow propagation and constraints are embedded if we formulate the dynamic DTA model basing on the cell transmission model (CTM), proposed by Daganzo (1995), which inherently considers link-capacity constraints. Further, the prescriptive model must take the flow departure rate from the origin zones as one of the decision variables in order to ensure an orderly or staged emergency evacuation.

The purpose of this paper is to establish such a many-to-one dynamic system optimal model and to provide an efficient solution method to this model. We develop a linear programming (LP) model for DSO based on CTM and the work by Ziliaskopoulos (2000) as the prescriptive DTA model for emergency evacuation. Although the original model in Ziliaskopoulos (2000) takes full advantage of CTM and the many-to-one network simplicity, the departure rate from origin zones are not controlled as part of system optimal strategies. However, both optimal routes and schedules are needed for an orderly emergency evacuation. Therefore the departure times at the origin zones will be taken as one of the control variables.

Efficient tools are needed to solve the LP model. The existing LP solution approach uses time-expanded networks to compute the system optimal and requires a user-provided upper bound on the evacuation time. However, due to the large number of the constraints, computational cost could be prohibitive with the existing solution approach, for a network of reasonable size. The number of constraints is highly related to the scale of the network and the number of discretized time intervals. When the network becomes large and the studying time is divided into short intervals, the total number of constraints will be huge. Therefore the conventional approach for solving this LP model suffers from high computational cost and may not scale up to large transportation networks, as being pointed out Ziliaskopoulos (2000).

On the other hand, it is unreasonable to consume long time to achieve high precision optimal solution, especially when the demands are implicit or network topology is changing. For those cases, certain approximations are preferred in order to generate the prescriptive dynamic traffic pattern quicker, because shorter computation time is more important in case of the real-time evacuation management. Therefore in this paper a **h**euristic **a**lgorithm for **s**taged **t**raffic **e**vacuation (HASTE) is being developed to provide a close approximation of the system optimum. HASTE can be used to find close-to-optimal evacuation strategies with reduced computational cost. It is useful for evacuation scenarios with large size networks and scenarios that do not require a high-precision optimal solution, but need to produce an efficient solution with a limited amount of time.

The remainder of this paper is organized as follows. Section 2 depicts the system objectives in emergency evacuation, and presents how the emergency evacuation can be formulated as a many-to-one DSO problem. Section 3 provides the detailed LP model with a cell-based network for emergency evacuation. In section 4, we propose the heuristic algorithm HASTE and provide computational cost analyses. In section 5, hypothetical evacuation examples are tested, and the performances of different approaches are compared. Finally, we summarize our findings in the last section.

2. EMERGENCY EVACUATION PROBLEM

The evacuation objectives for predictable disasters differ from those for unpredictable disasters, although both aim at minimizing the disaster-related losses. For predictable disasters, which mainly are natural disasters including hurricane, earthquake, tsunami, etc., evacuation is required even before disaster happens. Under this condition, the evacuation time window is known based on the scientific (e.g., meteorological, geological, or hydromechanical) forecasts. Thus, the primary evacuation objective under this situation is to minimize the system costs of evacuation, including total evacuation time and operation spending.

In contrast, emergency evacuation for unpredictable disasters, which mainly are man-made disasters or terrorist attacks, is urgently required right after the disaster occurrence time, and therefore is conducted in a much shorter time period, for example, only 55 minutes for people escaping from the South tower of the World Trade Center in September 11th terrorist attacks. Without a prediction to the disaster, the evacuation time window becomes short. The critical evacuation objective is replaced by minimizing the total evacuation time or minimizing the clearance time at which the last vehicle leaves the disaster-impacted areas.

To formulate the evacuation problem as an analytical model, we need to draw reasonable assumptions of evacuation scenario. We first assume that emergency evacuation happens under a no-notice disaster, which has a short evacuation time window. We also assume the disaster-impacted zones are known and the safe zones are given, as well as the number of evacuees. In an urgent evacuation operation, all evacuees are assumed to follow guidance such that the whole system achieves a global optimum, dynamically. The nature of emergency evacuation offers the possibility of implementing the strict traffic guidance and control. From the individual evacuee's point of view, he or she needs effective guidance to reach the safe areas as quickly as possible. Therefore, in case of evacuation, strict traffic controls, e.g. traffic guidance and control at major intersections, are not only feasible but also desirable. The purpose is to guide traffic under evacuation to evolve towards certain desired traffic states so that the designated system objectives can be achieved. With the aforementioned objective, the emergency evacuation problem should be modelled as a dynamic system optimal (DSO) problem.

This DSO problem can be further simplified as a many-to-one problem. Based on the consideration that all destinations essentially are the same for every evacuee, we adopt the concept of "super destination zone" proposed in Chiu *et al.* (2005). Technically, physical destinations can all be connected to one hypothetical "super node" by dummy links without travel costs. The "super destination zone" concept is exhibited as Figure 1.

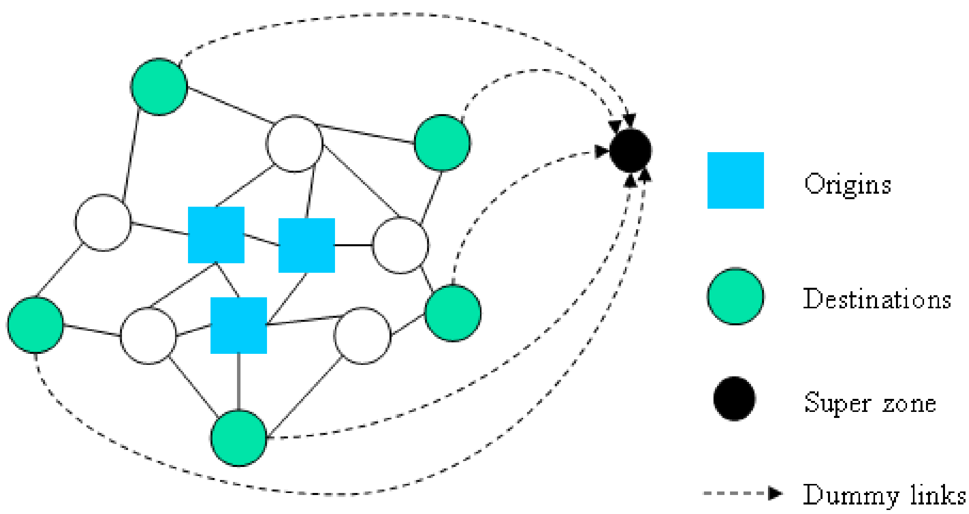

Figure 1 "Super destination zone" representation

3. MODEL

In this section, we develop a linear program (LP) model for the DSO problem using a cell-based network for emergency evacuation. For completeness, we first offer a brief review of the cell-transmission model (CTM) model. Notations that will be used in this paper are listed as follows:

q : link flow
q_{max} : maximum flow
k : density
k_J : jam density
v : link free flow speed

w : backward propagation speed
C : set of cells; $=\{C_O, C_D, C_M, C_R, C_S,\}$
C_O : ordinary cells set (with one predecessor and one successor)
C_D : diverging cells set (with one predecessor and multiple successors)
C_M : merging cells set (with multiple predecessors and one successor)
C_R : source cells set (without predecessor)
C_S : sink cells set (without successor)
E : set of cells connectors; $=\{E_O, E_D, E_M, E_R, E_S\}$
E_O : ordinary cell connectors set (connecting ordinary cells)
E_D : diverging cell connectors set (with a diverging cell head)
E_M : merging cell connectors set (with a merging cell tail)
E_R : source connectors set (connecting source cell)
E_S : sink connectors set (connecting sink cell)
T : set of discrete time intervals
x_i^t : number of vehicles in cell *i* at time interval *t*
y_{ij}^t : number of vehicles moving from cell *i* to cell *j* at time interval *t*
N_i^t : maximum number of vehicles can be held in cell *i* at time interval *t*
Q_i^t : maximum number of vehicles can flow into or out of cell *i* during interval *t*
d_i : total demand flow our of source cell *i*
d_i^t : demand flow out of source cell *i* during time interval *t*
δ_i^t : the ratio of *w/v* in cell *i* at time interval *t*
$\Gamma(i)$: set of successor cells to cell *i*
$\Gamma^{-1}(i)$: set of predecessor cells to cell *i*
ζ_i : initial number of vehicles presenting in cell *i*

3.1 Cell transmission model

The cell transmission model, proposed by Daganzo (1995), is a discretized transformation of kinematic wave model proposed in Lighthill and Whitham (1955) and Richard (1956). The CTM divides every link in the network into homogeneous cells, with a carefully-selected length that a vehicle traverses in one time interval by free-flow speed. And then, the LWR equations are approximated by a set of discretized difference equations.

The theories behind the CTM are two fundamental equations: the flow conservation equation and the flow-density relationship. The flow conservation equation represents the cell occupancy conservation where the occupancy of each cell at time t equals its occupancy at time $t-1$, plus the inflow and minus the outflow at time $t-1$, as follows:

$$x_i^t = x_i^{t-1} + \sum_{k\in\Gamma^{-1}(i)} y_{ki}^{t-1} - \sum_{j\in\Gamma(i)} y_{ij}^{t-1}, \qquad \forall i \in C, t \in T \tag{3.1}$$

Another equation is the triangular flow-density relationship (also shown in Figure 2). It states that the number of vehicles y_{ij}^t that traverse from upstream cell into the downstream cell must be restricted by (i) the number of vehicles x_i^t in the upstream cell *i* at time *t*, (ii) the maximum flow rate Q_i^t between these two cells, and (iii) the remaining occupancy at downstream cell which satisfies $\delta\left(N_j^t - x_j^t\right)$, where $\delta = 1$ if $x_{i-1}^t \le Q_i^t$, otherwise $\delta = w/v$, in which v and w are the free flow speed and the backward shockwave speed, respectively. Therefore, the flow-density relationship is presented by:

$$y_{ij}^t = \min\left\{x_i^t, Q_i^t, \delta\left(N_j^t - x_j^t\right)\right\}, \qquad \forall (i,j) \in E, t \in T \tag{3.2}$$

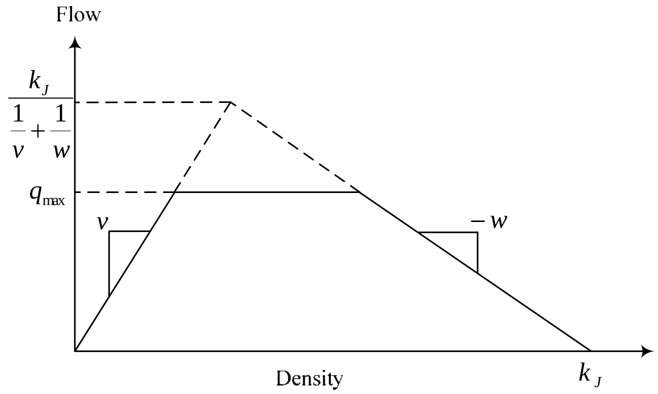

Figure 2 Triangular flow-density relationship in CTM

Daganzo (1995) also provided three categories of cells in the CTM: the ordinary cell with one upstream cell and one downstream cell; the diverging cell, which connects one upstream cell and multiple downstream cells; and merging cells having multiple upstream cells and one downstream cell. The three categories of cells follow different representations of the equations (3.1) and (3.2). Figure 3 demonstrates these three categories of cells:

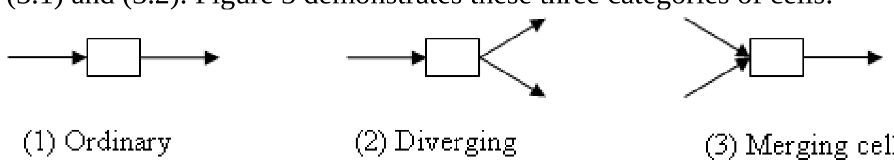

Figure 3 Classification of cells

3.2 Dynamic system optimal model with departure rate control

Based on the cell transmission model, Ziliaskopoulos (2000) formulated the single-destination system optimum DTA problem as a linear program. However, his model requires predetermined dynamic demands for each OD pair, which limit the application on evacuation. The departure rate control from origins can be utilized by the management authorities to help achieving the system optimal state. Therefore the departure rates d_i^t should be regarded as decision variables in the LP model and then determined by the optimal solution.

We modify Ziliaskopoulos' LP model by introducing the departure rates d_i^t as decision variables and corresponding demand conservation constraints. Consequently, other than the dynamic flow rates between cells, the solution of our LP model also provides the optimal departure rates d_i^t at origin zones. The dynamic system optimal model with departure rate control for emergency evacuation is listed as follows:

$$\min \quad \sum_{t\in T}\sum_{i\in C\setminus C_S} x_i^t + \sum_{t\in T}\sum_{i\in C_R} \beta_i d_i^t (t-1) \tag{3.3}$$

$$\text{s.t.} \quad x_i^t - x_i^{t-1} - \sum_{k\in\Gamma^{-1}(i)} y_{ki}^{t-1} + \sum_{j\in\Gamma(i)} y_{ij}^{t-1} = 0, \qquad \forall i\in C\setminus\{C_R, C_S\}, \forall t\in T \tag{3.4}$$

$$y_{ij}^t - x_i^t \le 0,\ y_{ij}^t \le Q_i^t,\ y_{ij}^t \le Q_j^t,\ y_{ij}^t + \delta_j^t x_j^t \le \delta_j^t N_j^t, \qquad \forall (i,j)\in E_O\cup E_R, \forall t\in T \tag{3.5}$$

$$y_{ij}^t - x_i^t \le 0,\ y_{ij}^t \le Q_i^t, \qquad \forall (i,j)\in E_S, \forall t\in T \tag{3.6}$$

$$y_{ij}^t \le Q_j^t,\ y_{ij}^t + \delta_j^t x_j^t \le \delta_j^t N_j^t, \qquad \forall (i,j)\in E_D, \forall t\in T \tag{3.7}$$

$$\sum_{\forall j\in\Gamma(i)} y_{ij}^t - x_i^t \le 0,\ \sum_{\forall j\in\Gamma(i)} y_{ij}^t \le Q_i^t, \qquad \forall i\in C_D, \forall t\in T \tag{3.8}$$

$$y_{ij}^t - x_i^t \le 0,\ y_{ij}^t \le Q_i^t, \qquad \forall (i,j)\in E_M, \forall t\in T \tag{3.9}$$

$$\sum_{\forall i\in\Gamma^{-1}(j)} y_{ij}^t \le Q_j^t,\ \sum_{\forall i\in\Gamma^{-1}(j)} y_{ij}^t + \delta_j^t x_j^t \le \delta_j^t N_j^t, \qquad \forall j\in C_M, \forall t\in T \tag{3.10}$$

$$x_i^t - x_i^{t-1} + y_{ij}^{t-1} - d_i^t = 0, \qquad \forall i\in C_R, \forall t\in T \tag{3.11}$$

$$d_i - \sum_{\forall t\in T} d_i^t = 0, \qquad \forall i\in C_R \tag{3.12}$$

$$x_i^0 - d_i^0 = 0, \qquad \forall i\in C_R \tag{3.13}$$

$$x_i^0 = \zeta_i, \qquad \forall i\in C\setminus\{C_R, C_S\} \tag{3.14}$$

$$y_{ij}^0 = 0, \qquad \forall (i,j)\in E \tag{3.15}$$

$$x_i^t \ge 0, \qquad \forall i\in C, \forall t\in T \tag{3.16}$$

$$y_{ij}^t \ge 0, \qquad \forall (i,j)\in E, \forall t\in T \tag{3.17}$$

Because the time interval τ is constant, minimizing the objective (3.3) is equivalent to minimizing the total evacuation costs for all cells over the studying time horizon T, by multiplying (3.3) the time interval τ. The entire evacuation costs for all evacuees are separated as two parts: the total travel time in the network, and the total waiting cost. By multiplying the time interval τ, the summation $\sum_{t\in T}\sum_{i\in C\setminus C_S} x_i^t$ represents the total system travel time of all vehicles. In the objective function, $\beta_i d_i^t (t-1)$ depicts that the waiting time of vehicles departing at time t is t-1, where parameters β_i are the waiting cost weights. Therefore, the summation $\sum_{t\in T}\sum_{i\in C_R} \beta_i d_i^t (t-1)$ represents the total waiting cost over the evacuation. Notice that the weights β_i are relevant to the locations, vehicle classification, or numbers of evacuees at the origins. For example, the weights could be higher for the origin zones close to the disaster location.

Constraint (3.4) shows the flow conservation on cells, which simply describes the relationships of the cell occupancies at two successive time intervals. Constraints (3.5) and (3.6) state at time interval t, the flow-density relationship, on source, ordinary, and sink cell connectors, respectively. Constraints (3.7) and (3.8) regulate the flow out of diverging cell i at time interval t. Constraints (3.9) and (3.10) regulate the amount of flow transmitted into merging cell i at time interval t. Constraint (3.11) describes the departure rate control, at source origin cell i at time interval t. Constraint (3.12) states the demand conservation, and (3.13) provides the initialization at origin cells. Equation (3.14) represents the initial number of vehicles at each cell. Constraint (3.15) initializes the traffic flow status. Finally, (3.16) and (3.17) are non-negativity constraints.

Note that in the LP model, the study horizon T must be predetermined as a constant. It is the best to select the study horizon T as the network clearance time, during which all travellers have finished their trips. Without prior knowledge, it may be difficult to select such an optimal value. If a shorter horizon T is selected, sub-optimal solutions may be achieved due to the lack of the information on the future conditions. On the other hand, there exist useless variables and constraints if a bigger T is used.

As indicated in Ziliaskopoulos (2000), the CTM-based DSO model is not suitable to solve problem with large network size. The model contains a large number of variables and constraints due to the multiplication of number of links, number of cell on each links and the number of discretized time intervals. When the network scale becomes larger and the studying time is shorter, both numbers of variables and constraints are huge and computational time is prohibitive using the existing solution approaches. Considering these disadvantages in the LP model, we introduce a heuristic approach in next section to approximate the dynamic system optimum.

4. HEURISTIC SOLUTION METHODS

In this section, we first discuss the complexities of existing LP solvers, and then develop an alternative heuristic approach, HASTE, to approximate the optimal solution.

4.1 Linear program solvers

Linear program solvers are provided in many scientific software packages. Generally, they apply either simplex method, or interior point method. Dantzig in 1947 developed the famous simplex method in a tableau form. Although the complexity of simplex methods has been proven to be exponential in worst case, they generally have good practical performances which were presented in literature. It has been proven that the average computational cost of primal dual simplex methods is $O((n-m)mn^2)$, where n is the number of variables and m is the number of constraints in the problem. Therefore, simplex methods are still widely accepted in practice.

In contrast, the interior point method, firstly developed by Khachyan (1979), is to achieve a polynomial computation time and to overcome the possible exponential complexity of simplex method. Originally, the method has a computational complexity of $O(n^4L)$, where L is the bit length of data in computer. However, the state-of-the-art algorithm that is widely applied in the software packages (e.g., CPLEX, LOQO, OSL) reduces its computational complexity as $O(n^3L)$, and even $O\left(\left(n^3/\log n\right)L\right)$ given by Anstreicher (1999).

Undoubtedly, the most straightforward and convenient solution approach to the DSO model presented above is to apply the LP solvers provided in commercial software. However, the huge number of variables in our LP model require a long computational time. As a prescriptive DTA model to offer a real-time guidance for emergency evacuation, the DSO model needs a short computational time which is much more important than an exact optimal solution. Therefore, it is important to seek an alternative solution approach, which provides a close-to-optimum solution with highly reduced computational cost.

4.2 HASTE

We develop a **h**euristic **a**lgorithm for **s**taged **t**raffic **e**vacuation, or HASTE in short, as an approach to approximate the system optimum. To reduce the system travel time, the basic idea in HASTE is that through the departure rate control, travellers will use the same facilities at different times to avoid delay. Available link capacity on the shortest path should be fully utilized, but cannot be exceeded. Therefore capacities along the links on dynamic shortest paths need to be reserved for different evacuee group at different times.

Before the main iteration, a sorting procedure needs to be performed to determine the order of origins by which all evacuees are assigned the time-dependent shortest paths in the main iteration. This order, denoted as $\hat{R}$, can be established by different criteria related to origins, such as the straight distances from the origins to the border of the safe area, the number of evacuees in the origins, or importance of evacuees in the origins. The order $\hat{R}$ will be maintained throughout the HASTE iterations as long as there are evacuees left at origins.

The detailed implementation of HASTE is described in the flow chart, as shown in the Figure 4. The pseudo code of HASTE is shown in the Appendix. For each iteration, origins are processed according to the order $\hat{R}$, and the time-dependent shortest path p from current non-empty origin to the "super zone" is searched. The number of evacuees f_p, who depart at the time t through the same path p, is computed by the minimal capacity, or bottleneck, along path p. The capacities along path p are then reserved by this group of evacuees and the demand at the origin is reduced by f_p. To be consistent with the cell transmission model, the available capacity on each link is restricted by the remaining occupancy of downstream cell at preceding time interval. When all evacuees are assigned their departure time and evacuation routes, the iteration terminates. Note that the study horizon T is self-determined in HASTE, in contrast of a predetermined one required for LP solvers.

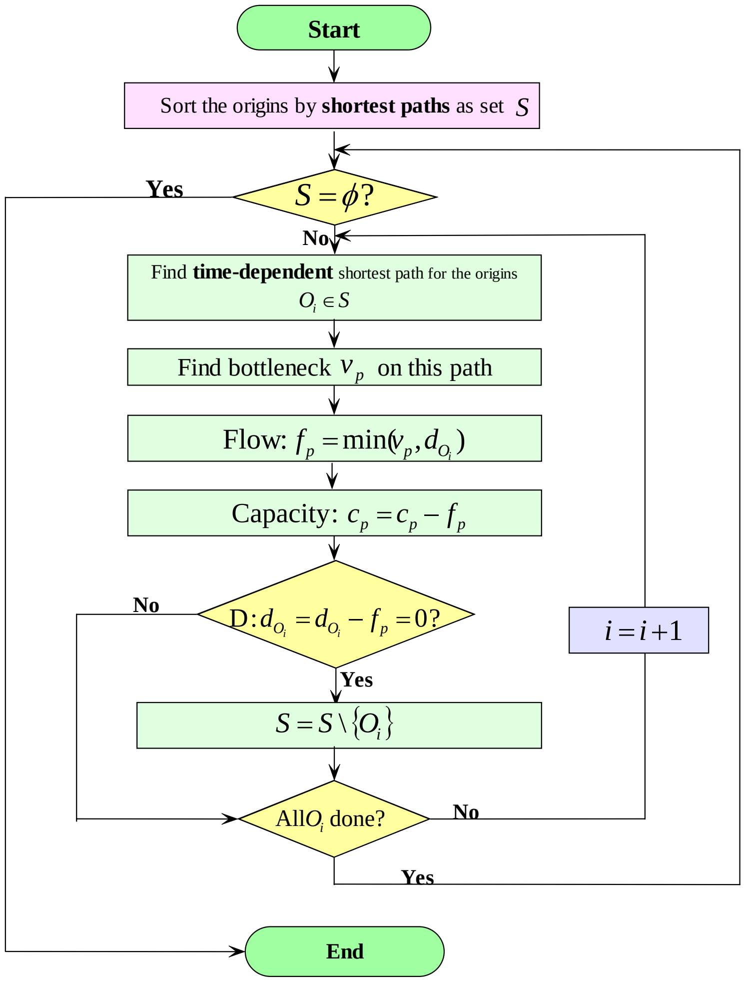

Figure 4. Flow chart of HASTE Algorithm

HASTE applies a time-dependent shortest path search. The queuing delay is considered based on the dynamic traffic flow that has been assigned into the network. Whenever the downstream cell has been fully occupied at the evacuees' arrival time *t*, evacuees must wait at current cell until downstream cell has usable space to traverse through.

The number of shortest path search of HASTE is the same as the number of evacuees groups, N_g, each of which contains the evacuees who traverse through the same path and depart at the same time. Path p guarantees at least one evacuee travelling through it to arrive at the destination, and therefore, in the worst case, each group only contains one individual evacuee. As a result, the number of iterations has an upper bound equal to the number of evacuees. In another word, if we assume that M is the total number of evacuees, then $N_g = O(M)$.

At each iteration, the computational costs contain two main parts: the time-dependent shortest path search and the capacities reservation. The time-dependent shortest path search is implemented by generalizing Dijkstra's shortest path algorithm. In addition, the capacities reservation is implemented on all the cells and cell-connectors over the study time intervals T.

Since the capacities reservation is only done on a small subset of cells and cell-connectors, the shortest path search dominates the computation time in HASTE.

If the HASTE applies the time-dependent shortest path search based on a link-based network in contrast to a cell-based network, we may have two different algorithms with distinct computational complexities. We discuss the two different implementations as follows.

4.2.1 HASTE-I

The first possible implementation that HASTE perform shortest path search over the cell-based network $G(C, E)$. And hence the number of nodes in generalized Dijkstra's shortest path algorithm is the number of cells over study horizon T in the given cell-based network. Therefore, the number of variables in the model is $n = |C| * |T|$.

In HASTE-I, the number of iterations $N_g = O(M)$. At each iteration, the computational cost is the summation of the computational cost of Dijkstra's shortest path algorithm, $O(n \log n)$ and the network loading cost $O(n)$. Therefore, the overall computational complexity of HAST-I is $O((n \log n + n)N_g)$, or $O((n \log n + n)M)$.

4.2.1 HASTE-II

In HASTE-II, time-dependent shortest path search is applied on the original link-based network $G(N, L)$, instead of the discretized cell-based one $G(C, E)$. This is reasonable in practice since only intersections could be under control, instead of cells. Therefore, the number of nodes in generalized Dijkstra's algorithm is the number of nodes N multiplied by the study horizon T. Let $k = |N| * |T|$ is the number of variables in the shortest path search. Then $k << n$, due to the fact that the number of nodes N is much smaller than the number of cells C.

The number of iterations in HASTE-II N'_g can be different from the iteration number of HASTE-I; however, the scale keeps the same. Thus, $N'_g = O(M)$. The computational cost of Dijkstra's shortest path algorithm reduces from $O(n \log n)$ to $O(k \log k)$. The evacuees are still assigned on the cell-based network $G(C, E)$, such that the network loading cost is the same as HASTE-I, as $O(n)$. Therefore, the overall computational complexity of HASTE-II is $O((k \log k + n)N'_g)$, or $O((k \log k + n)M)$. Since the number of nodes generally is much smaller than the number of cells (i.e., $k << n$), the computing time of HASTE-II is faster than HASTE I. However, since the most congested cell will be used to represent the link for the capacity reservation in HASTE-II, the solution of HASTE-II may not be so close to the optimal one as that of HASTE-I.

5. NUMERICAL EXAMPLES

To demonstrate the applicability of the LP model and the heuristic for the system optimal evacuation problem with departure time control, we apply both LP solver and HASTE to a small network and a hypothetical evacuation scenario with relatively large network, and compare their performances using both examples.

5.1 A small test network

For convenience, we employ a simple network that has seven nodes, eight links, with two origins A and B, and one destination G. The network is similar with the example used in Ziliaskopoulos (2000), but with two origin zones instead of one. The network structure is shown as Figure 5, with the characteristics of the simple network shown in Table 1.

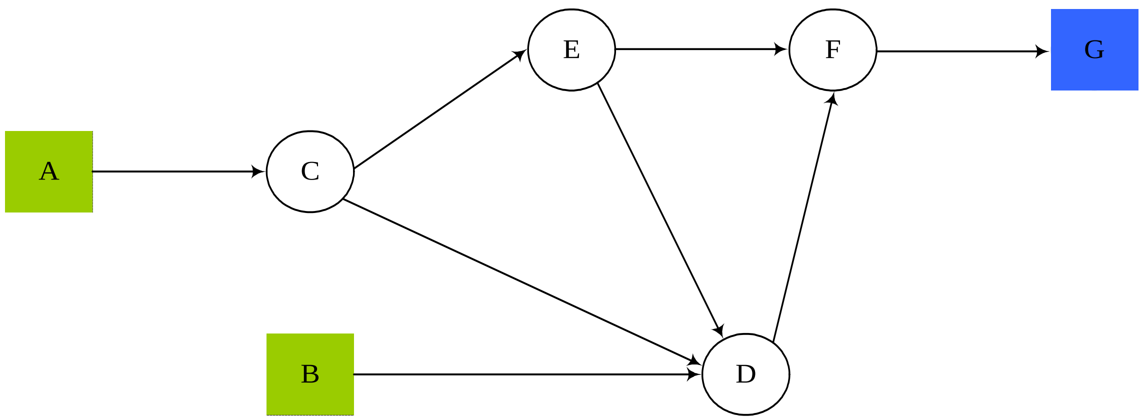

Figure 5 A small test network

Table 1 Characteristics of the simple network

Arcs	AC	BD	CD	CE	ED	EF	DF	FG
Length(ft)	500	500	1000	500	500	500	1000	500
# of lanes	2	2	1	1	1	1	1	2
Spd lmt (ft/s)	50	50	50	50	50	50	50	50
Max flow (vph)	4320	4320	2160	2160	2160	2160	2160	4320

The studying time period of 120 seconds has been equally divided into twelve small time intervals, each of which contains ten seconds. By the given free flow speed and the length for each link in the simple network, the example network can be divided as cells represented by the following Figure 6.

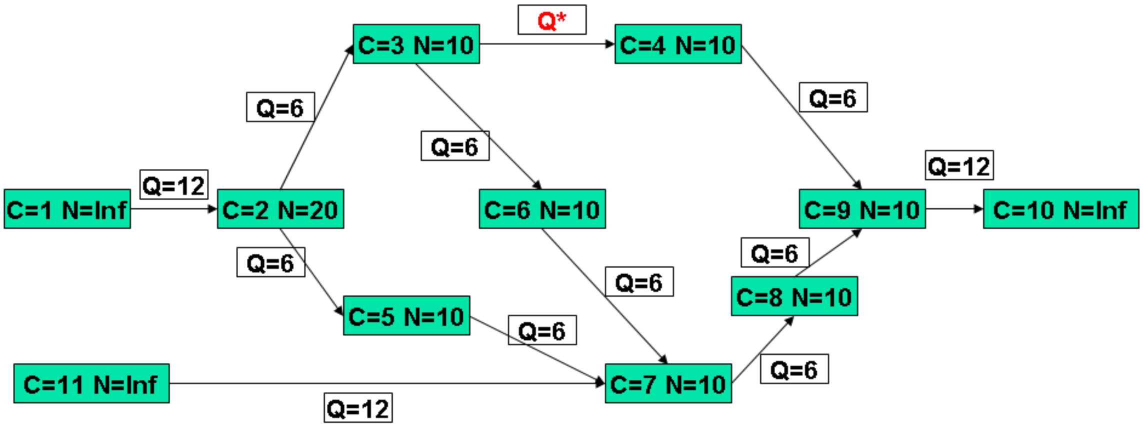

Figure 6 Cell representation of the simple network

In the cell representation, the cell 1 and 11 are origins; the cell 10 is the single destination; cells 4, 5, 6, and 8 are ordinary cells; cells 2 and 3 are diverging cells; cells 7 and 9 are merging cells. The maximal flow rates between cells are determined by the product of max flow rate and the 10-second studying interval. In addition, certain incident on link EF makes the max flow rate between cells 3 and 4 varies along with time, as shown in the Table 2.

Table 2 Time dependent max flow rate for cell connector (3, 4)

Time	1	2	3	4	5	6	7	8	9	10	11	12
Q*=	6	6	0	0	3	3	6	6	6	6	6	6

We implement the system optimal DTA LP model for the simple network in a program with 300 variables and 546 constraints. We directly apply the linear programming solver offered by MATLAB using a primal-dual interior-point algorithm to solve this LP problem. The computational results are shown in Table 3. The arrival rates at destination cell are 6, 4, 9, 7, 12, 8, 12, and 6, for time 4 to 11, respectively. The time dependent flow restrictions on cell connector (3, 4) shown in Table 2 are also represented by the occupancies of cell 4 shown in Table 3. Although the cell connector (3, 4) is on the shortest path, there is no flow traversing this connector until time interval 5. In addition, the queuing constraints, $y_{ij}^t \le N_j^t - x_j^t$, are demonstrated by occupancies of cells 7 and 8 from time 2 to time 8, where the cell occupancies are restricted by remaining capacities of downstream cells at previous time interval. The total evacuation time or the optimal objective value is 428.

We also implement HASTE to solve on the same problem. The solution of time dependent cell occupancies is shown in Table 4. All departure rates are easily observed by the cell occupancies at origin cells 1 and 11. And the solution is demonstrated as a close approximation to the optimal one, resulting in higher total system time of 433. The shade cells in these two tables show the difference of their arrival rates at these time intervals.

Table 3 Optimal time dependent cell occupancies provided by LP solver

Cell #	Time											
	1	2	3	4	5	6	7	8	9	10	11	12
1	19.3	15.3	12.7	10.3	7.8	5.2	1.9	0	0	0	0	0
11	11.5	11.4	11.4	8.9	8	4.9	3.5	0.9	0	0	0	0
2	0	8.9	8.2	8.3	8.4	7.1	7	2.7	0	0	0	0
3	0	0	4	5.5	5.2	6.4	4	6	2.7	0	0	0
4	0	0	0	0	3	3	6	4	6	2.7	0	0
5	0	0	2.2	2.7	2.9	2.8	2.4	1.1	0	0	0	0
6	0	0	0	1.9	2.2	2.1	1.6	0.8	0	0	0	0
7	0	6	4	6	4	6	4	6	3.3	0	0	0
8	0	0	6	4	6	4	6	4	6	3.3	0	0
9	0	0	0	6	4	9	7	12	8	12	6	0

Table 4 Time dependent cell occupancies provided by the HASTE

Cell #	Time											
	1	2	3	4	5	6	7	8	9	10	11	12
1	10	6	4	12	0	0	0	0	0	0	0	0
11	10	10	0	3	0	3	1	5	0	0	0	0
2	0	10	6	4	12	0	0	0	0	0	0	0
3	0	0	6	4	6	4	6	0	0	0	0	0
4	0	0	0	0	3	3	6	4	3	0	0	0
5	0	0	0	0	0	6	0	0	0	0	0	0
6	0	0	0	3	4	0	0	0	0	0	0	0
7	0	10	0	10	0	9	1	9	1	5	0	0
8	0	0	6	4	6	4	6	4	6	4	5	0
9	0	0	0	6	4	9	7	12	8	9	4	5

Both LP solver and HASTE have very short computational time, due to the small scale of the problem, such that the comparisons of computational efficiency between both approaches become insignificant. To demonstrate the computational efficiency of HASTE I/II and compare the complexity differences between LP solvers and HASTE, we develop a hypothetical evacuation problem based on a practical network, which is presented in next section.

5.2 A Hypothetical Scenario with Large Network

5.2.1 Network description

The hypothetical evacuation problem assumes that a no-notice terrorist attack happens in the downtown Minneapolis, MN. The area within a one-mile-diameter cycle is the potential disaster-impacted region, in which all people must be evacuated within 45 minutes.

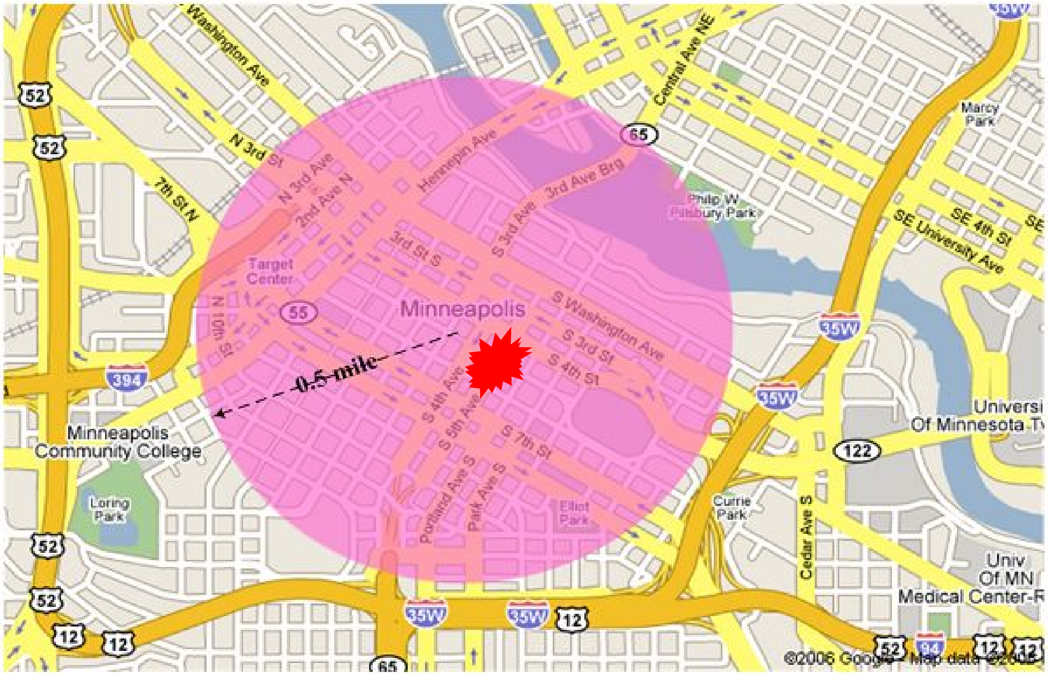

Figure 7 Hypothetical disaster impacted area

The map of downtown Minneapolis and the disaster-impacted area are shown as Figure 7. We extract the disaster-impacted area and get an abstract network as shown in Figure 8, in which the origin zones aggregate as centroids. This abstract network encloses 172 nodes, 469 links and 17 origins with 15977 evacuees. The data is provided by the Metropolitan Council of the Twin Cities based on the pm peak data in Year 2000 planning model. All necessary characteristics of the links, including the free flow speeds, capacities, lengths, number of lanes, and jam density, are provided in the data set.

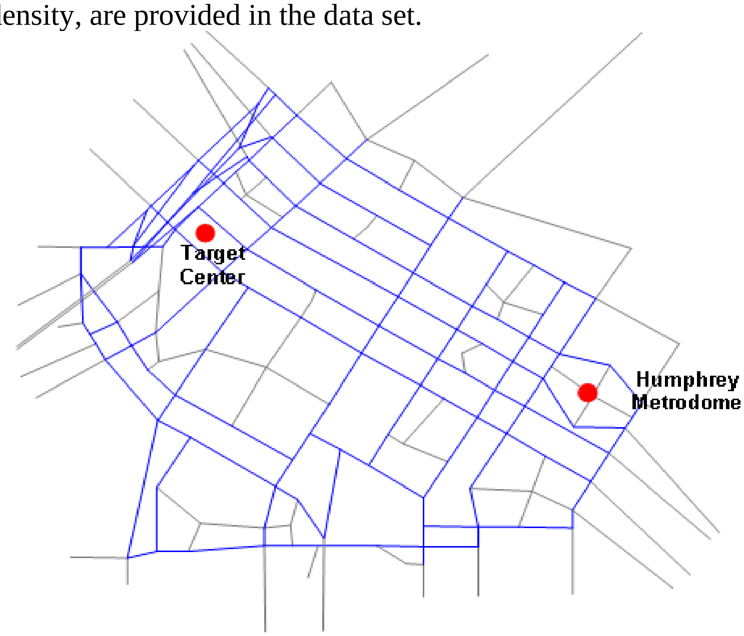

Figure 8 Evacuation network representation

5.2.2 Cell representation

Before we apply the DSO model and solution methods to the problem, we need to transform the network in to a cell-based one. Firstly, we assume the length of each studying interval is 15 seconds. The network has a cell representation with 935 cells, 1627 cell-connectors and 180 studying time intervals. It is noteworthy that few links in the network can be perfectly transformed into integer number of cells, because the physical length of link and its free flow speed may not perfectly fit the studying time interval of 15 seconds. The perfect case as the small example does not exist in practice. As a result, the number of cells has been rounded up when the value is not an integer.

5.2.3 Results analysis

We organize the variables and constraints matrix of the DSO LP model with departure rate in a software package GAMS (Brooke *et al*., 1998) and directly apply the LP solver provided by CPLEX, which utilizes the primal-dual interior-point algorithm with $O(n^3L)$ computational complexity. The LP model for the half-mile-radius hypothetical network finally contains 461161 variable and 1466428 constraints in CPLEX.

We also implement the HASTE I and HAST II by MATLAB, following the steps shown in Section 4. All the computational implementations are conducted on a personal computer with 3 GHz CPU and 2 GB ram. Solutions of LP solver, HASTE I and HASTE II are combined in Table 5.

Table 5 Solutions to the hypothetical evacuation problem

	CPLEX 9.0	HASTE-I	HASTE-II
Computational complexity	$O(n^3L)$	$O((n\log n+n)N_g)$	$O((k\log k+n)N_g)$
Objective value	1138278	1171220	1245456
Clearance time	00:36:15	00:39:45	00:41:45
CPU time (sec.)	14149	7883	2159

From the computational results, although the evacuation time window is 45 minutes, the evacuation completes after 36 minutes and 15 seconds, resulting in hundreds of thousands useless variables and constraints existing in the LP model. However, this clearance time can not be known before we implement the program. Therefore, the unused variables and constraints will always exist in the LP model, which obviously affect the computational efficiency in LP solvers. In contrast, the HASTE expands the evacuation time by itself according to the time-dependent shortest path search in each evacuee group assignment. Although the objective value and clearance time are higher than the optimal solution, both HASTE approaches provide their solutions mush faster than the LP solver. For an easier comparison, we put the differences between different approaches all together into Table 6.

Table 6 Differences between different approaches

	CPLEX 9.0		
	Objective value	Clearance time	CPU time
HASTE-I	+2.89%	+9.66%	-44.29%
HASTE-II	+9.42%	+15.17%	-84.74%

HASTE-I takes about half computational time to provide a solution only with only 3 percent higher objective value than the optimal one, and about 10 percent higher in terms of clearance time. The HASTE-II severely reduces the computational cost. It takes only 15 percent computational time of CPLEX and provides the approximation about 10 percent higher than the optimal solution in terms of objective value, and 15 percent higher in terms of clearance time. Based on the comparisons, HASTE-II outperforms the other two approaches in terms of computational time and HASTE-I compromises the solution precision and the algebra complexity.

With regard to the CPU time of these three approaches, the implementation of HASTE-I and HASTE-II may be further improved, since MATLAB is based on script which is less efficient that other programming languages, such as C or java. Therefore, the CPU time of HASTE-I and HASTE-II can be lower than the value listed in Table 5.

6. CONCLUSIONS

In this paper, we propose a many-to-one dynamic system optimal model for the real-time traffic operation under emergency evacuation. The DSO model is formulated as a linear problem by using a cell-based network representation. Departure rate control is considered in the model as decision variables to ensure an efficient evacuation. Due to the high computational cost for solving the LP model, we also develop a heuristic algorithm HASTE to provide close-to-optimum solution to the model in a lower computational cost which is important to real-time evacuation operations. Numerical examples demonstrate that using the heuristic algorithm can achieve significant reduction of computational cost with reasonable accuracy.

REFERENCES

Anstreicher K. M. (1999). Linear programming in $O\left(\left(n^3/\log n\right)L\right)$ operations. *SIAM Journal of Optimization*, **9**, 803-812.

Akamatsu, T. (2000). A dynamic traffic equilibrium assignment paradox. *Transportation Research B*, **34**(6), 515-531.

Brooke, A., D. Kendrick and A. Meeraus (1998). GAMS, a user's guide. GAMS Development Corporation.

Chiu, Y.C., P. Koraka and P.B. Mirchandani (2005). Dynamic traffic management for evacuation. *Transportation Research Record*, 05-2369.

Daganzo, C.F. (1995). The Cell Transmission Model, Part II: Network Traffic. *Transportation Research B*, **29**(2), 79-93.

Khachyan, L. G. (1979). A polynomial algorithm in linear programming. Translated in *Soviet Mathematics Doklady*, **20,** 191--194.

Kuwahara, M. and T. Akamatsu (1993). Dynamic Equilibrium Assignment with Queues for a One-to-Many OD Pattern. *Transportation and Traffic Theory*, **12**, 185-204.

Lighthill, M.J., and G.B. Whitham (1955). On Kinematic Waves: I. Flow modeling in long rivers. II. A theory of traffic flow on long crowded roads. *Proc. Roy. Soc. A*, **229**, 281-345.

Liu, H.X., J.X. Ban, W. Ma and P. Mirchandani (2006). Model reference adaptive control framework for real time traffic management under emergency evacuation. ASCE Journal of Urban Planning and Development, in press.

Richards, P.I. (1956). Shock waves on the highway. *Operations Research*, **4**, 42-51.

Southworth, F. (1991). Regional Evacuation Modeling: A State of the Art Review. Centre for transportation analysis, Oak Ridge National Laboratory, A report prepared for the U.S. Department of the Army.

Urbina, E., and B. Wolshon (2003). National review of hurricane evacuation plans and policies: A comparison and contrast of state practices, Transportation Research, 37A(3), 257-275.

Ziliaskopoulos, A.K. (2000). A linear programming model for the single destination system optimum dynamic traffic assignment problem. *Transportation Science,* **34**, 37-49.

Appendix: Pseudo Code of HASTE

Input:

(1): $G(C, E)$: a graph G with a set of cells C and a set of cell connectors E

Each cell $i \in C$ at time $t \in T$ has three properties:

Maximum_Cell_Occupancy, N_i^t : non-negative

Maximum_Cell_Capacity, Q_i^t : non-negative

Current_Cell_Occupancy, x_i^t : non-negative

Each cell connector $(i, j) \in E$ at time $t \in T$ has three properties:

Upstream_Cell_Index, i : non-negative

Downstream_Cell_Index, j : non-negative

Current_Flow, y_{ij}^t : non-negative

(2): R : set of origins, $C_R \subseteq C$

(3): S : destination ("super zone"), $C_S \in C$

(4): d_i : set of demands on each origin cell $i \in C_R$

Output: Evacuation plan: dynamic shortest routes with number of evacuees on each route

Method:

Pre-process: sort the origins as $\hat{R}$ in an order of their weights α_r, $\forall r \in R$; (A0)

assign the initial cell occupancy $x_i^0 = \zeta_i$, $\forall i \in C$;

Available_Cell_Occupancy $(i, t) = N_i^t$;

Available_Connector_Capacity $(i, j, t) = \min(Q_i^t, Q_j^t)$;

while any evacuee still in any origin, in an order of $\hat{R}$, do { (A1)

if *demand* $d_i > 0$, do {

find the time-dependent shortest path $p : < i_0, i_1, ..., i_n >$ with time schedule

$< t_0, t_1, ..., t_n >$, such that *Available_Cell_Occupancy*: $occu(i_k, t_k)$ >0 and

Available_Connector_Capacity: $capa(i_k, i_{k+1}, t_k)$ >0; (A2)

find the bottleneck v = min($occu(i_k, t_k)$, $capa(i_k, i_{k+1}, t_k)$, $\forall k \in \{0,1,...n\}$); (A3)

find the *flow* = min(d_i ,v); (A4)

for $k = 0 : n-1$ do { (A5)

Available_Cell_Occupancy: $occu(i_k, t_k) = occu(i_k, t_k)$ -*flow*; (A6)

Available_Connector_Capacity: $capa(i_k, i_{k+1}, t_k) = capa(i_k, i_{k+1}, t_k)$ -*flow*; (A7)

$x_{i_k}^{t_k} = x_{i_k}^{t_k} + flow$; (A8)

$y_{i_k i_{k+1}}^{t_k} = y_{i_k i_{k+1}}^{t_k} + flow$; (A9)

}

demand $d_i = d_i - flow$; (A10)

}

}

Output evacuation plan. (A11)