1. (10 pts.) Perform, with paper and pencil, the following computations using 4-digit, base 10 arithmetic. Each computation is to be performed using the “rounding to closest”. Rounding should be performed at each step of the calculation. For each computation discuss the results in terms of both absolute and relative errors. In these problems \( w = 7.235 \times 10^{-4}, x = 2.455 \times 10^0, y = 8.420 \times 10^{-4}, \) and \( z = 1.877 \times 10^3 \).

a) \((w + y) + z\)

b) \((z + y) + w\)

c) \(\frac{y}{z}\)

d) \(\frac{z}{w}\)

e) \(x \times z\)

2. (20 pts.) Let \( f(x) = \sin^2(x) \).

a) Construct MATLAB functions to approximate the derivative of \( f(x) \) using the following two formulae:

\[
D_1[f(x)] = \frac{f(x + h) - f(x)}{h},
\]

and

\[
D_2[f(x)] = \frac{f(x + h) - f(x - h)}{2h}.
\]

Both your MATLAB function, \( D_1[f(x)] \) and \( D_2[f(x)] \) should accept \( x \) and \( h \) as arguments and produce an approximate derivative as output. Also construct a MATLAB function to evaluate the exact derivative \( f'(x) \).

b) Calculate the relative errors for the approximate derivatives from \( D_1[f(x)] \) and \( D_2[f(x)] \) at the points \( x = -0.8 \) and \( x = 0 \). Use step sizes given by \( h = 10^{-n} \) where \( n = 1, 2, \ldots, 10 \). Create a table for each value of \( x \) with the headings \( h \), \( D1error \), and \( D2error \).

c) For each value of \( x \), what value of \( h \) yields the most accurate approximation?

d) (5 extra points) Explain the difference in the behavior of the errors at \( x = 0 \) versus \( x = -0.8 \).
3. (15 pts.) Write a MATLAB code to approximate the solutions to the quadratic equation, 
\[ ax^2 + bx + c = 0 \] using the quadratic formula 
\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \] for the following cases

a) \( a = 2, \ b = 3, \ c = 1 \)
b) \( a = 1, \ b = 2, \ c = -2 \)
c) \( a = 10^{-12}, \ b = 3, \ c = 3. \)

Keeping floating-point arithmetic in mind, discuss the accuracy of the computed result with respect to the exact roots. For any roots that are inaccurate, present and implement a more accurate method for computing the solution. Comment on your results.

Hint, the roots for the three cases are given below to 17-digits of accuracy

a) \( x_1 = -5.000000000000000 \times 10^{-1}, \ x_2 = -1.000000000000000 \times 10^{0} \)
b) \( x_1 = 7.3205080756887729 \times 10^{-1}, \ x_2 = -2.7320508075688773 \times 10^{0} \)
c) \( x_1 = -1.000000000003333 \times 10^{0}, \ x_2 = -2.999999999990000 \times 10^{12}. \)