

# Analytical Fully-Recursive Sensitivity Analysis for Multibody Dynamic Chain Systems

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## Abstract

This paper presents a novel fully recursive method, a direct differentiation based approach, which facilitates first-order sensitivity analysis in optimal design problems involving multibody dynamic systems. A state space  $O(n)$  dynamic analysis algorithm based on a velocity space projection method, as promoted by Kane [?], forms the foundation of the underlying formulation. This algorithm can significantly reduce the massive number of mathematical and associated computational operations involved in explicitly generating and solving the sensitivity equations. This benefit is particularly evident for systems involving a combination of many state variables and design parameters. The development presented in this paper focuses on chain systems to illustrate the recursive nature of the algorithm. The computational efficiency and solution accuracy of the presented algorithm are investigated through the procedure's application to the simulation and design sensitivity determination of spatial chain systems involving 2, 4, 6,  $\dots$ , 24 degrees of freedom, as well as a simple planar double pendulum.

Key words: multibody optimization, sensitivity analysis, recursive algorithm, Order- $n$

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